Slides week 37

Overview of some natural occurring distributions

Independent trials Register: A/A^c $P(A)=p$	Events in disjoint timeintervals are independent $P(\text{One event in } \Delta t)=\lambda\Delta t+o(\Delta t)$ $P(\text{More than one event in } \Delta t)=o(\Delta t)$
X=number of times A occurs in n trials $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}, x=0,1,\ldots,n$	X=number of times A occur in [0,t] $P(X=x)=\frac{(\lambda t)^x e^{-\lambda t}}{x!}, x=0,1,2,\ldots$
X=number of trials until A occurs for the first time $P(X=x)=(1-p)^{x-1}p, \text{ x}=1,2,\ldots$	X= time until A occurs for the first time $f_X(x)=\begin{cases}\lambda e^{-\lambda x}, & \text{x>0}\\ 0, & \text{otherwise}\end{cases}$
X=number of trials until A occurs for the r-th time $P(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}, \text{ x}=r,r+1,\ldots$	X=time until A occurs the r-th time $f_X(x)=\begin{cases}\frac{\lambda^r}{\Gamma(r)}x^{r-1}e^{-\lambda x}, & \text{x>0}\\ 0, & \text{otherwise}\end{cases}$

Gamma distribution

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}},\ x > 0,\ \alpha>0,\ \beta>0.$$

$$X \sim \Gamma(\alpha,\beta) \Rightarrow Y = cX \sim \Gamma(\alpha, c\beta)$$

$$E\left[X^n\right] = \frac{\Gamma(\alpha+n)\beta^n}{\Gamma(\alpha)},\ n > \text{-}\alpha$$

$$\alpha = 1 \Rightarrow X \sim \exp\left(\frac{1}{\beta}\right)$$

$$\alpha = \frac{\nu}{2},\ \beta=2 \Rightarrow \mathrm{X} \sim \chi^2(\nu)$$

$$X_i \sim \Gamma(\alpha_i, \beta), i = 1, 2, \ldots, n \Rightarrow \sum_{i=1}^{n} X_i \sim \Gamma\left(\sum_{i=1}^{n} \alpha_i, \beta\right)$$

Beta distribution

$$f\left(x\middle|\alpha,\beta\right)=\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}x^{\alpha-1}\left(1-x\right)^{\beta-1},\ 0<x<1,\ \alpha>0,\ \beta>0$$

$$E\left[X^{n}\right]=\frac{\Gamma\left(\alpha+n\right)\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha+\beta+n\right)\Gamma\left(\alpha\right)},\ n>-\alpha$$