Exercise 10 TMA4295

Problem 1

Let $X_1, \ldots, X_n$ be i.i.d. Poisson(λ). We shall estimate

$$\tau(\lambda) = e^{-\lambda}$$

(which is the probability of a single X being 0).

a) Show that the MLE of λ is $\bar{X}$ (which should be well known).

b) Find expected value and variance of the following estimators of $\tau(\lambda)$.

1. $e^{-\bar{X}}$ (which is the MLE of $\tau(\lambda)$. Why?)
2. $(1 - \frac{1}{n})^{n\bar{X}}$

(*Hint:* Let $T = \sum_{i=1}^{n} X_i = n\bar{X}$, so that $T \sim \text{Poisson}(n\lambda)$. Why? It will be helpful to express both estimators in terms of T, so that, e.g., estimator 2 is $(1 - \frac{1}{n})^T$. Then you may either proceed directly, or use the moment generating function $M_T(t)$ (which is well known), in order to to compute the expected values and variances).

Problem 2

7.38

Problem 3

7.47

Problem 4

6.18

Problem 5

7.52