Exercise 9 TMA4295

Problem 1

7.2.a from the book

Problem 2

7.24 from the book

Problem 3

7.40 from the book

Problem 4

7.41 from the book

Problem 5

Let $\boldsymbol{X} = (X_1, \ldots, X_n)$ be n independent observations from $N(\theta, \sigma^2)$, where σ^2 is known.

a) Find Cramer-Rao's lower bound for the variance of unbiased estimators for θ. (This was done in class).

b) Is $\bar{X}$ an UMVUE for θ?

c) Show that Cramer-Rao's lower bound for the variance of unbiased estimators for θ^2 is

$$\frac{4\theta^2\sigma^2}{n}$$

d) Show that

$$W(\boldsymbol{X}) = \bar{X}^2 - \frac{\sigma^2}{n}$$

is an unbiased estimator of θ^2, but has a variance that is larger than Cramer-Rao's lower bound. (*Hint:* In order to simplify the computation

You may use that the fourth order moment in a normal distribution is:

$$E\left[X^4\right] = \theta^4 + 6\theta^2\sigma^2 + 3\sigma^4.$$

Problem 6

In the derivation of the Cramer-Rao inequality we assumed that the parameter θ is one-dimensional. By going through the proof one may see that there may well be more parameters than θ in the modell. If, for example, θ, η are unknown, we may deduce

$$Var_{\theta,\eta}(W) \geq \frac{(\frac{d}{d\theta} E_{\theta,\eta} W)^2}{Var_{\theta,\eta}(\frac{\partial}{\partial\theta} \log f(\boldsymbol{X}|\theta))}$$

Let $X_1, \dots, X_n$ be i.i.d. from $N(\mu, \sigma^2)$, where both parameters are unknown. Find lower bounds on the variance of unbiased estimators of, respectively, μ and σ^2. Compare to the variances of $\bar{X}$ and S^2.