Bivariate transformations

Monotone

$$\begin{aligned} U &= g_1(X,Y) \\ V &= g_2(X,Y) \end{aligned} \Rightarrow \begin{cases} X = h_1(U,V) \\ Y = h_2(U,V) \end{cases}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$f_{U,V}(u,v) = f_{X,Y}\left(h_1(u,v), h_2(u,v)\right)|J|$$

Hierarchical Models and Mixture Distributions

$$X|Y \sim B(Y,p)$$

$$Y|\Lambda \sim \mathrm{Po}(\Lambda)$$

$$\Lambda \sim \exp(\beta)$$

$$E[X] = E\left[E[X|Y]\right]$$

$$Var[X] = E\left[Var[X|Y]\right] + Var\left[E[X|Y]\right]$$