Week 39

Hølders Inequality

$$\left|E[XY]\right| \le E|XY| \le \left(E|X|^p\right)^{\frac{1}{p}} \left(E|X|^q\right)^{\frac{1}{q}}, \quad \frac{1}{p}+\frac{1}{q}=1$$

Jensen's Inequality

$$E[g(X)] \ge g\left(E[X]\right), \; g(x) \text{ convex}$$

Chapter 5 Random Sample

Random sample: $X_1,\dots,X_n$ are iid.

Statistic: $T(X_1,\dots,X_n)$

<u>Some properties of Statistics</u>

$X_1,\dots,X_n$ are $N(\mu,\sigma^2)$

$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2$ are independent

$$\bar{X} \sim N\left(\mu,\frac{\sigma^2}{n}\right), \quad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

T-statistic: $\dfrac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$, In general $T_p=\dfrac{N(0,1)}{\sqrt{\frac{\chi^2(p)}{p}}}$

$$Var\left[T_p\right]=\frac{p}{p-2}$$