Repetition week 41

Delta method

$$\sqrt{n}\left(Y_n-\theta\right)\xrightarrow{D}N\left(0,\sigma^2\right)\Rightarrow\sqrt{n}\left(g\left(Y_n\right)-g\left(\theta\right)\right)\xrightarrow{D}N\left(0,\sigma^2\left[g'(\theta)\right]^2\right)$$

$$g'\left(\theta\right)=0$$

$$\sqrt{n}\left(Y_n-\theta\right)\xrightarrow{D}N\left(0,\sigma^2\right)\Rightarrow n\left(g\left(Y_n\right)-g\left(\theta\right)\right)\xrightarrow{D}\frac{\sigma^2}{2}\left[g''(\theta)\right]\chi_1^2$$

Sufficient statistics

A statistic $T\left(\boldsymbol{X}\right)$ is a sufficient statistic for $\boldsymbol{\theta}$ if the conditional distribution of the sample $\boldsymbol{X}$ given the value of $T\left(\boldsymbol{X}\right)$ does not depend on $\boldsymbol{\theta}$.

A sufficient statistics for a parameter (-vector) $\boldsymbol{\theta}$ is a statistic that in a certain sense, captures all the information about $\boldsymbol{\theta}$ in the sample.

Theorem 6.2.2

If $p(\boldsymbol{x}|\boldsymbol{\theta})$ is the pdf/pmf of $\boldsymbol{X}$ and $q(t|\boldsymbol{\theta})$ is the pdf/pmf of $T(\boldsymbol{X})$, then $T(\boldsymbol{X})$ is a sufficient statistics for $\boldsymbol{\theta}$ if, for every $\boldsymbol{x}$ in the sample space the ratio $\dfrac{p(\boldsymbol{x}|\boldsymbol{\theta})}{q(T(\boldsymbol{x})|\boldsymbol{\theta})}$ is a constant as a function of $\boldsymbol{\theta}$.

Theorem 6.2.6

Let $f(\boldsymbol{x}|\theta)$ be the joint pdf/pmf for a sample $\boldsymbol{X}$. $T(\boldsymbol{X})$ is a sufficient statistics for θ if and only if for all $\boldsymbol{x}$ and all θ.

$$f(\boldsymbol{x}|\theta) = g(T(\boldsymbol{x})|\theta)h(\boldsymbol{x})$$