Repetition week 42

Minimal sufficient.

Definition 6.2.11. A sufficient statistics $T(\boldsymbol{X})$ is called a minimal sufficient statistics if for any other sufficient statistics $T^{'}(\boldsymbol{X})$, $T(\boldsymbol{X})$ is a function of $T^{'}(\boldsymbol{X})$.

Theorem 6.2.3

Let $f(\boldsymbol{x}|\theta)$ be the joint pdf/pmf for a sample $\boldsymbol{X}$. Suppose there exists a $T(\boldsymbol{X})$ such that for every $\boldsymbol{x}$ and every $\boldsymbol{y}$, $f(\boldsymbol{x}|\theta)/f(\boldsymbol{y}|\theta)$ is a constant as a function of $\theta \Leftrightarrow T(\boldsymbol{X})=T(\boldsymbol{Y})$. Then $T(\boldsymbol{X})$ is a minimal sufficient statistics for θ.

Definition 6.2.21

Let $f(t|\theta)$ be a family of pdfs/pmfs for a statistic $T(\boldsymbol{X})$. The family is complete if

$E_\theta\left[g(T)\right]=0 \Rightarrow P_\theta\left(g(T)=0\right)=1$, for all θ.

Point Estimation

Moment estimators

$$\mu_1\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\vdots$$

$$\mu_k\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right) = \frac{1}{n}\sum_{i=1}^{n} x_i^k$$

Maximum likelihood estimation

Likelihood:

$$L(\theta|\boldsymbol{x}) = f(\boldsymbol{x}|\theta) \overset{iid}{=} \prod_{i=1}^{n} f(x_i|\theta)$$

$\hat{\theta}_e(\boldsymbol{x})$ maximizes $L(\theta|\boldsymbol{x})$

$\hat{\theta}(\boldsymbol{X})$ is the MLE

Candidates: For $\hat{\theta}$

$$\frac{\partial}{\partial\theta_i} L(\boldsymbol{\theta}) = 0 \Leftrightarrow \frac{\partial}{\partial\theta_i} \log L(\boldsymbol{\theta}) = 0$$