Slides week 37

Exponential Class of distributions

$$f\left(x\middle|\boldsymbol{\theta}\right)=h\left(x\right)c\left(\boldsymbol{\theta}\right)e^{\sum_{i=1}^{k}w_i(\boldsymbol{\theta})t_i(x)}$$

$$E\left(\sum_{i=1}^{k}\frac{\partial w_i\left(\boldsymbol{\theta}\right)}{\partial\theta_j}t_i\left(X\right)\right)=-\frac{\partial}{\partial\theta_j}\log c\left(\boldsymbol{\theta}\right)$$

$$Var\left(\sum_{i=1}^{k}\frac{\partial w_i\left(\boldsymbol{\theta}\right)}{\partial\theta_j}t_i\left(X\right)\right)=-\frac{\partial^2}{\partial\theta_j}\log c\left(\boldsymbol{\theta}\right)-E\left(\sum_{i=1}^{k}\frac{\partial^2 w_i\left(\boldsymbol{\theta}\right)}{\partial^2\theta_j}t_i\left(X\right)\right)$$

Location – Scale Families

$f\left(x\right)$ pdf. The family of pdfs: $\frac{1}{\sigma}f\left(\frac{x-u}{\sigma}\right)$,

$\mu\in\left(-\infty,\infty\right),\ \sigma>0$

The distribution of $Y=\mu+\sigma X$

Chebyshevs

$g\left(x\right)\geq 0,\ r>0$

$$P\left(g\left(X\right)\geq r\right)\leq\frac{Eg\left(X\right)}{r}$$

Bivariate transformations

Monotone

$$
\begin{aligned} U &= g_1(X,Y) \\ V &= g_2(X,Y) \end{aligned} \Rightarrow \begin{cases} X = h_1(U,V) \\ Y = h_2(U,V) \end{cases}
$$

$$
J = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}
$$

$$
f_{U,V}(u,v) = f_{X,Y}\left(h_1(u,v), h_2(u,v)\right)|J|
$$