Slides Week 40

Convergence in distribution

$\{X_i\}_{i=1}^{\infty} \xrightarrow{D} X$ if $\lim_{n\to\infty} F_{X_n}(x) = F_X(x)$ at all x where $F_X(x)$ is continuous.

$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X \Rightarrow \{X_i\}_{i=1}^{\infty} \xrightarrow{D} X$

Central Limit Theorem

$\{X_i\}_{i=1}^{\infty}$ iid, $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2 < \infty$.

Define $X_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then $\sqrt{n}\left(\frac{X_n - \mu}{\sigma}\right) \xrightarrow{D} X$ where $X \sim \mathrm{N}(0,1)$.

Slutsky's Theorem.

$X_n \xrightarrow{D} X$, $Y_n \xrightarrow{P} a$, then

a) $X_n Y_n \xrightarrow{D} aX$

$b) X_n + Y_n \xrightarrow{D} X + a$

Delta method

$$\sqrt{n}\left(Y_n - \theta\right) \xrightarrow{D} N\left(0, \sigma^2\right) \Rightarrow \sqrt{n}\left(g\left(Y_n\right) - g\left(\theta\right)\right) \xrightarrow{D} N\left(0, \sigma^2\left[g'(\theta)\right]^2\right)$$