Repetition week 43

Bayes estimation:

Prior: $\pi(\theta)$ Posterior: $\pi(\theta|x)$

$$\pi(\theta|x)=\frac{f(x,\theta)}{f(x)}=\frac{f(x|\theta)\pi(\theta)}{\int f(x,\theta)d\theta}$$

$$\hat{\theta}_B = E(\theta|x)$$

The mean square error

$$MSE = E\left[(W-\theta)^2\right] = Var[W]+(E[W]-\theta)^2$$

Score statistics

$$S(X|\theta)=\frac{\partial}{\partial\theta}\log f(X|\theta)$$

$$E\left[S(X|\theta)\right]=0$$

Let $\tau(\theta)=E[W(X)]$

Cramer-Rao

$$Var[W(X)]\geq\frac{\left(\frac{\partial}{\partial\theta}\tau(\theta)\right)^2}{I_X(\theta)}$$

Equality

If and only if $S(X|\theta)=a(\theta)[W(X)-\tau(\theta)]$