Slides week 36

Exponential Class of distributions

$$f(x|\boldsymbol{\theta}) = h(x)c(\boldsymbol{\theta})e^{\sum_{i=1}^{k} w_i(\boldsymbol{\theta})t_i(x)}$$

$$E\left(\sum_{i=1}^{k} \frac{\partial w_i(\boldsymbol{\theta})}{\partial \theta_j} t_i(X)\right) = -\frac{\partial}{\partial \theta_j} \log c(\boldsymbol{\theta})$$

$$Var\left(\sum_{i=1}^{k} \frac{\partial w_i(\boldsymbol{\theta})}{\partial \theta_j} t_i(X)\right) = -\frac{\partial^2}{\partial \theta_j} \log c(\boldsymbol{\theta}) - E\left(\sum_{i=1}^{k} \frac{\partial^2 w_i(\boldsymbol{\theta})}{\partial^2 \theta_j} t_i(X)\right)$$

Location – Scale Families

$f(x)$ pdf. The family of pdfs: $\frac{1}{\sigma} f\left(\frac{x-u}{\sigma}\right)$,

$\mu \in (-\infty, \infty)$, $\sigma > 0$

The distribution of $Y = \mu + \sigma X$

Chebyshevs

$g(x) \geq 0$, r>0

$$P(g(X) \geq r) \leq \frac{Eg(X)}{r}$$

Bivariate transformations

Monotone

$$\begin{aligned} U &= g_1(X,Y) \\ V &= g_2(X,Y) \end{aligned} \Rightarrow \begin{cases} X = h_1(U,V) \\ Y = h_2(U,V) \end{cases}$$

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$$

$$f_{U,V}(u,v) = f_{X,Y}\left(h_1(u,v), h_2(u,v)\right)|J|$$