TMA4295 Statistical inference
Exercise 7 - solution

Problem 6.3
We want to use the factorization theorem (6.2.6), so let's look at the joint pdf

$$f(x_1,\ldots,x_n|\mu,\sigma)=\prod_{i=1}^n \frac{1}{\sigma}e^{\frac{-(x_i-\mu)}{\sigma}}I_{(\mu,\infty)}(x_i)=\left(\frac{e^{\mu/\sigma}}{\sigma}\right)^n e^{-\sum_i x_i/\sigma}I_{(\mu,\infty)}(\min(x_1,...,x_n))$$

so $T(\mathbf{X})=(\min_i(X_i),\sum_i X_i)$ is a sufficient statistics for (μ,σ).

Problem 6.6
We want to use the factorization theorem, so let's look at the joint pdf

$$f(x_1,\ldots,x_n|\alpha,\beta)=\prod_{i=1}^n \frac{1}{\Gamma(\alpha)\beta^\alpha}x_i^{\alpha-1}e^{-x_i/\beta}=\frac{1}{\Gamma(\alpha)^n\beta^{n\alpha}}\left(\prod_{i=1}^n x_i\right)^{\alpha-1}e^{-\sum_i x_i/\beta}$$

so $T(\mathbf{X})=(\prod_i(X_i),\sum_i X_i)$ is a sufficient statistics for (μ,σ).

Problem 6.9

b) Here we will use theorem 6.2.13 so let's look at the ratio of pdf

$$\frac{f(x_1,\ldots,x_n|\theta)}{f(y_1,\ldots,y_n|\theta)}=\frac{\prod_{i=1}^n e^{-(x_i-\theta)}I_{(\theta,\infty)}(x_i)}{\prod_{i=1}^n e^{-(y_i-\theta)}I_{(\theta,\infty)}(y_i)}=\frac{e^{n\theta}e^{-\sum_i x_i}\prod_{i=1}^n I_{(\theta,\infty)}(x_i)}{e^{n\theta}e^{-\sum_i y_i}\prod_{i=1}^n I_{(\theta,\infty)}(y_i)}=\frac{e^{-\sum_i x_i}I_{(\theta,\infty)}(\min_i x_i)}{e^{-\sum_i y_i}I_{(\theta,\infty)}(\min_i y_i)}.$$

The ratio will be independent of θ if and only if $\min_i(x_i)=\min_i(y_i)$, then $\min_i(X_i)$ is a minimal sufficient statistics.

Problem 4

a) $X_1,...,X_n\sim Unif(0,\theta)$ iid. Using the factorization theorem we have

$$f(x_1,...,x_n|\theta)=\prod_{i=1}^n \frac{1}{\theta}I_{(0,\theta)}(x_i)=\left(\frac{1}{\theta}\right)^n I_{(0,\infty)}(\min_i(x_i))I_{(0,\theta)}(\max_i(x_i)).$$

Then $T(\mathbf{X})=\max_i(x_i)$ is a sufficient statistics for θ.

b) $X_1,...,X_n\sim Unif(-\theta,\theta)$ iid. Using the factorization theorem we have

$$f(x_1,...,x_n|\theta)=\prod_{i=1}^n \frac{1}{2\theta}I_{(-\theta,\theta)}(x_i)=\prod_{i=1}^n \frac{1}{2\theta}I_{[0,\theta)}(|x_i|)=\left(\frac{1}{2\theta}\right)^n I_{[0,\theta)}\max_i(|x_i|).$$

Then $T(\mathbf{X})=\max_i(|x_i|)$ is a sufficient statistics for θ.

Problem 5
Here we will use theorem 6.2.13 so let's look at the ratio of pdf

$$\frac{f(x_1,\ldots,x_n|\theta)}{f(y_1,\ldots,y_n|\theta)}=\frac{p^{\sum_i x_i}(1-p)^{n-\sum_i x_i}}{p^{\sum_i y_i}(1-p)^{n-\sum_i y_i}}=\frac{p^{\sum_i x_i}(1-p)^{-\sum_i x_i}}{p^{\sum_i y_i}(1-p)^{-\sum_i y_i}}$$

The ratio will be independent of n if and only if $\sum_i x_i=\sum_i y_i$, so if and only if $\frac{1}{n}\sum_i x_i=\frac{1}{n}\sum_i y_i$, then $\frac{1}{n}\sum_i X_i$ is a minimal sufficient statistics.