Slides Week 39

Convergence concepts

Convergence in probability:

$$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X \text{ if } \forall\ \varepsilon > 0,\ \lim_{n\to\infty} P\left(\left|X_n - X\right| \geq \varepsilon\right) = 0.$$

Weak law of large numbers

$$\{X_i\}_{i=1}^{\infty}\ iid,\ \mathrm{E}\left[X_i\right] = \mu \text{ and } \mathrm{Var}\left(X_i\right) = \sigma^2 < \infty. \text{ Then } \lim_{n\to\infty} P\left(\left|\overline{X}_n - \mu\right| < \varepsilon\right) = 1$$

Convergence in distribution

$$\{X_i\}_{i=1}^{\infty} \xrightarrow{D} X \text{ if } \lim_{n\to\infty} F_{X_n}(x) = F_X(x) \text{ at all } x \text{ where } F_X(x) \text{is continuous.}$$

$$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X \Rightarrow \{X_i\}_{i=1}^{\infty} \xrightarrow{D} X$$

Central Limit Theorem

$$\{X_i\}_{i=1}^{\infty}\ iid,\ \mathrm{E}\left[X_i\right] = \mu \text{ and } 0<\mathrm{Var}\left(X_i\right) = \sigma^2 < \infty.$$

$$\text{Define } X_n = \frac{1}{n}\sum_{i=1}^{n} X_i. \text{ Then } \sqrt{n}\left(\frac{X_n - \mu}{\sigma}\right) \xrightarrow{D} X \text{ where } X \sim \mathrm{N}(0,1).$$

Slutsky's Theorem.

$X_n \xrightarrow{D} X$, $Y_n \xrightarrow{P} a$, then

a) $X_n Y_n \xrightarrow{D} aX$

$b) X_n + Y_n \xrightarrow{D} X + a$