# Repetition week 41

## Minimal sufficient.

Definition 6.2.11. A sufficient statistics $T(\mathbf{X})$ is called a minimal sufficient statistics if for any other sufficient statistics $T'(\mathbf{X})$, $T(\mathbf{X})$ is a function of $T(\mathbf{X})$.

## Theorem 6.2.3

Let $f(x|\theta)$ be the joint pdf/pmf for a sample $\mathbf{X}$. Suppose there exists a $T(\mathbf{X})$ such that for every $\boldsymbol{x}$ and every $\boldsymbol{y}$, $f(x|\theta)/f(y|\theta)$ is a constant as a function of $\theta \Leftrightarrow T(\mathbf{X})=T(\mathbf{Y})$. Then $T(\mathbf{X})$ is a minimal sufficient statistics for $\theta$.

Maximum likelihood estimation

*Likelihood:*

$$L(\theta|\boldsymbol{x}) = f(\boldsymbol{x}|\theta) \overset{iid}{=} \prod_{i=1}^{n} f(x_i|\theta)$$

$\hat{\theta}_e(\boldsymbol{x})$ maximizes $L(\theta|\boldsymbol{x})$

$\hat{\theta}(\boldsymbol{X})$ is the MLE

Candidates: For $\hat{\theta}$

$$\frac{\partial}{\partial\theta_i} L(\boldsymbol{\theta}) = 0 \Leftrightarrow \frac{\partial}{\partial\theta_i} \log L(\boldsymbol{\theta}) = 0$$

Invariance principle:

If $\hat{\theta}$ is the MLE of $\theta$, $\tau(\hat{\theta})$ is the MLE of $\tau(\theta)$.

$$f(\boldsymbol{x}|\theta) = f\left(\boldsymbol{x}\middle|\theta = \tau^{-1}(\eta)\right) \Rightarrow f\left(\boldsymbol{x}\middle|\hat{\theta}\right) = f\left(\boldsymbol{x}\middle|\hat{\theta} = \tau^{-1}(\hat{\eta})\right) = f\left(\boldsymbol{x}\middle|\hat{\theta} = \tau^{-1}\left(\tau\left(\hat{\theta}\right)\right)\right)$$

In case of $\tau(\theta)$ is not one to one:

$$L^*(\eta|\boldsymbol{x}) = \sup_{\{\theta:\tau(\theta)=\eta\}} L(\theta|\boldsymbol{x})$$