

Repetition week 42

Bayes estimation:

Prior: $\pi(\theta)$ Posterior: $\pi(\theta|x)$

$$\pi(\theta|x) = \frac{f(x, \theta)}{f(x)} = \frac{f(x|\theta)\pi(\theta)}{\int f(x, \theta)d\theta}$$

$$\hat{\theta}_B = E(\theta|x)$$

The mean square error

$$MSE = E[(W - \theta)^2] = Var[W] + (E[W] - \theta)^2$$

Score statistics

$$S(X|\theta) = \frac{\partial}{\partial \theta} \log f(X|\theta)$$

$$E[S(X|\theta)] = 0$$

$$\text{Let } \tau(\theta) = E[W(X)]$$

Cramer-Rao

$$Var[W(X)] \geq \frac{\left(\frac{\partial}{\partial \theta} \tau(\theta)\right)^2}{I_x(\theta)}$$

Equality

$$\text{If and only if } S(X|\theta) = a(\theta)[W(X) - \tau(\theta)]$$