Repetition week 46

Methods of Construction Confidence intervals

Inverting a test $H_0 : \theta = \theta_0 \quad H_1 : \theta \neq \theta_0$

$$A(\theta_0) = \{ \boldsymbol{x} : \boldsymbol{x} \in R^C \}$$

$$C(\boldsymbol{x}) = \{ \theta_0 : \boldsymbol{x} \in A(\theta_0) \}$$

Inverting LRT

$$C(\boldsymbol{x}) = \{ \theta_0 : \lambda(\boldsymbol{x}) \geq \mathrm{k} \}$$

Pivotal Quantity

The distribution of $Q(\boldsymbol{X}, \theta)$ is independent of θ.

$$C(\boldsymbol{x}) = \{ \theta : \alpha_1 \leq F_T(t|\theta) \leq 1 - \alpha_2 \}$$

Bayesian intervals

Credible sets.

$$P(\theta \in A|\boldsymbol{x}) = \int_A \pi(\theta|\boldsymbol{x}) d\theta$$

An example

$$X_1,\ldots X_n \text{ iid Poisson}(\lambda) \Rightarrow Y = \sum_{i=1}^{n} X_i \sim \text{Poisson}(n\lambda)$$

$$\pi(\lambda) = \text{gamma}(\alpha,\beta)$$

$\pi\left(\lambda \middle| \sum_{i=1}^{n} x_i = y\right) = \text{gamma}\left(\alpha + y, \frac{\beta}{n\beta+1}\right)$ which gives the $1-\alpha$ credibility interval

$$P\left(\frac{\beta}{2(n\beta+1)}\chi\left(2(y+\alpha)\right)_{1-\frac{\alpha}{2}} \le \lambda \le \frac{\beta}{2(n\beta+1)}\chi\left(2(y+\alpha)\right)_{\frac{\alpha}{2}}\right) = 1-\alpha$$

Which can be compared to the $1-\alpha$ confidence interval.

$$P\left(\frac{1}{2n}\chi(2y)_{1-\frac{\alpha}{2}} \le \lambda \le \frac{1}{2n}\chi\left(2(y+1)\right)_{\frac{\alpha}{2}}\right) = 1-\alpha$$

Credibility reflects subjective beliefs about uncertainties updated with the data.

Confidence intervals reflects uncertainty in the mechanism of repeated experiments.

Consistent estimator

$$\hat{\theta}(X_1,\ldots X_n) \xrightarrow{P} \theta, \ \forall \theta.$$