The EM-algorithm

- Situation:
  - observed data: x
  - likelihood function:
  $$L(\theta|x) = f_X(x|\theta) = \int f_{X,Z}(x,z|\theta)\mathrm{d}z$$
  where we have a formula for $f_{X,Z}(x,z|\theta)$
  - assume: no formula for $f_X(x|\theta)$, or not able to maximise $f_X(x|\theta)$ w.r.t. θ analytically
  - assume: can analytically maximise $f_{X,Z}(x,z|\theta)$ w.r.t. θ
  - want MLE for θ.
- EM-algorithm:
  - start with some intital value for θ: $\theta^{(0)}$
  - iterate for $t = 0, 1, 2, \ldots$ until convergence
    1. define
    $$Q(\theta|\theta^{(t)}) = \mathrm{E}\left[\ln f_{X,Z}(x,z|\theta)|x,\theta^{(t)}\right]$$
    2. set
    $$\theta^{(t+1)} = \operatorname*{argmax}_{\theta} Q(\theta|\theta^{(t)})$$