Access to the computer lab

- For those who do not have access to Nullrommet 380A: please send Ingeborg Hem (ingeborg.hem@ntnu.no) a mail as soon as possible with your name, NTNU username/e-mail address and study programme.
- You will get:
  - Physical access to the computer lab
  - User at the (linux) computers
- Don't wait, do it today!

Partner for the exercises

- The exercises should be done in groups of two persons. You can
  - let the teaching assistants find exercise partner for you. If so, send an email with your name and email address to Maxime Conjard (maxime.conjard@ntnu.no) and ask for this.
  - find an exercise partner yourself. If so, send an email with (the two) names and email addresses to the teaching assistant (maxime.conjard@ntnu.no).
- Don't wait, do it today. Deadline: Tuesday January 16th.

Reference group

- Three students should be in the reference group.
  - Preferably from different study programmes.
- Duties:
  - Stay in dialogue with the other students.
  - Participate in three meetings with the lecturer and the teaching assistants, and provide suggestions for improvement in exercises, lectures, home page, and so on.
  - Write a joint brief final report providing constructive feedback and evaluation of the course. This report will be published unedited in the course evaluation.
- Volunteers?

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  - pseudocode for simulating from $f_{X_1}(x_1)$:

    generate $x_2 \sim f_{X_2}(x_2)$
    generate $x_1 \sim f_{X_1|X_2}(x_1|x_2)$
    return x_1