

Metropolis–Hastings setup (discrete distributions)

- ▶ Target distribution: $f(x), x \in \Omega$ (given by problem)
- ▶ Proposal distribution: $Q(y|x), x, y \in \Omega$ (your choice)
- ▶ Acceptance probability: (given by algorithm)

$$\alpha(y|x) = \min \left\{ 1, \frac{f(y)}{f(x)} \cdot \frac{Q(x|y)}{Q(y|x)} \right\}$$

- ▶ Note: The defined Markov chain must be irreducible, aperiodic and positiv recurrent

Metropolis–Hastings setup (discrete distributions)

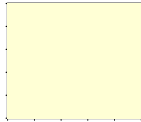
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- ▶ Note: The defined Markov chain must be irreducible, aperiodic and positiv recurrent
- ▶ Questions to answer:
 - ▶ how to construct such a Markov chain?
 - ▶ how to simulate the Markov chain?
 - ▶ how to find the value of m ?
 - ▶ how to estimate $\text{Var}[\hat{\mu}]$?

Ising model ($\beta = 0.87, x^0 = 0$)

$0n$ iterations

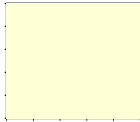


$200n$ iterations

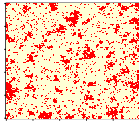
$400n$ iterations

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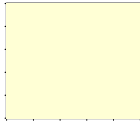
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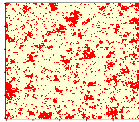
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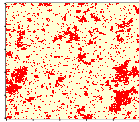
$0n$ iterations



$200n$ iterations

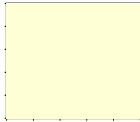


$400n$ iterations

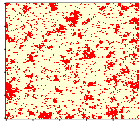


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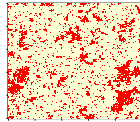
$0n$ iterations



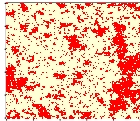
$200n$ iterations



$400n$ iterations



$600n$

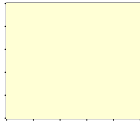


$800n$ iterations

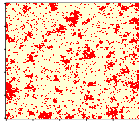
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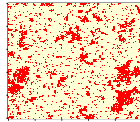
$0n$ iterations



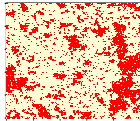
$200n$ iterations



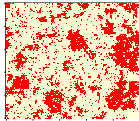
$400n$ iterations



$600n$



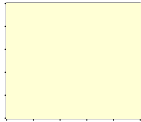
$800n$ iterations



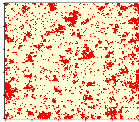
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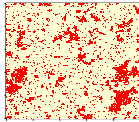
$0n$ iterations



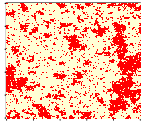
$200n$ iterations



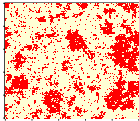
$400n$ iterations



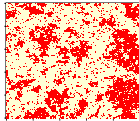
$600n$



$800n$ iterations

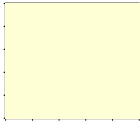


$1000n$ iterations

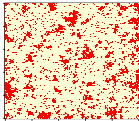


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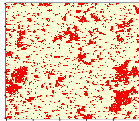
$0n$ iterations



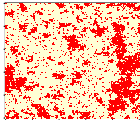
$200n$ iterations



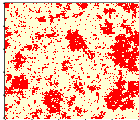
$400n$ iterations



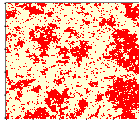
$600n$



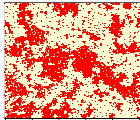
$800n$ iterations



$1000n$ iterations



$5000n$

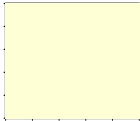


$10000n$ iterations

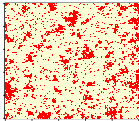
$15000n$ iterations

Ising model ($\beta = 0.87, x^0 = 0$)

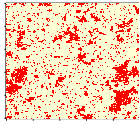
$0n$ iterations



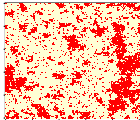
$200n$ iterations



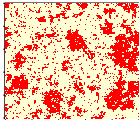
$400n$ iterations



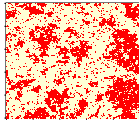
$600n$



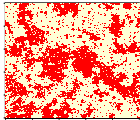
$800n$ iterations



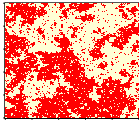
$1000n$ iterations



$5000n$



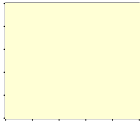
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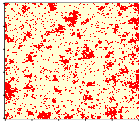
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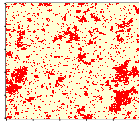
$0n$ iterations



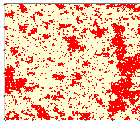
$200n$ iterations



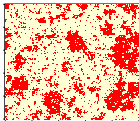
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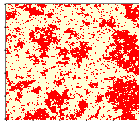
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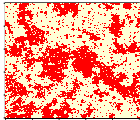
$800n$ iterations



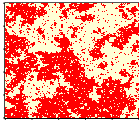
$1000n$ iterations



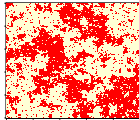
$5000n$



$10000n$ iterations

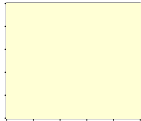


$15000n$ iterations

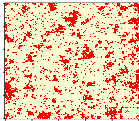


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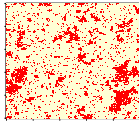
$0n$ iterations



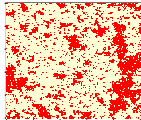
$200n$ iterations



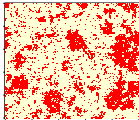
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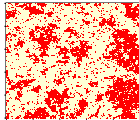
$600n$



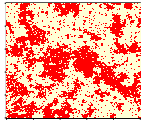
$800n$ iterations



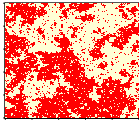
$1000n$ iterations



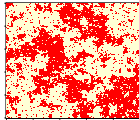
$5000n$



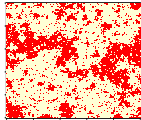
$10000n$ iterations



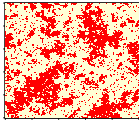
$15000n$ iterations



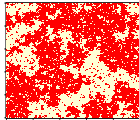
$20000n$



$25000n$ iterations

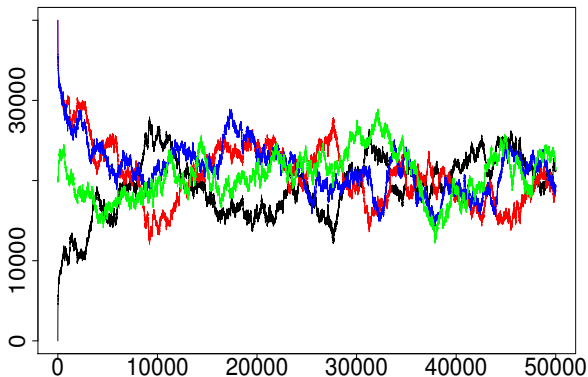


$30000n$ iterations



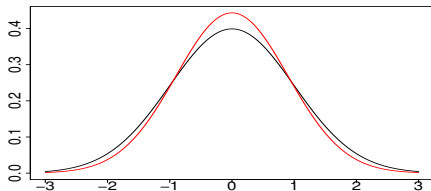
Ising model

- ★ Trace plot of number of 1's
 - four runs
 - different initial states:
 - + all 0's
 - + all 1's (two runs)
 - + independent random in each node

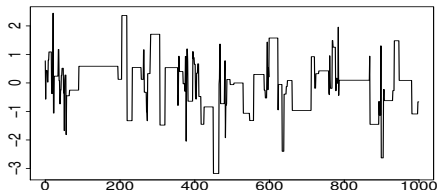


Toy example: Independent proposals

- ★ Target distribution: $x \sim N_{250}(0, I)$
- ★ Proposal distribution: $y|x \sim N_{250}(0, 0.9^2 \cdot I)$

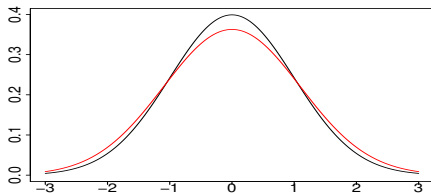


- ★ Trace plot of x^1

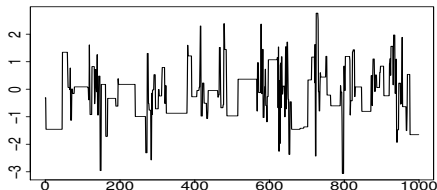


Toy example: Independent proposals

- ★ Target distribution: $x \sim N_{250}(0, I)$
- ★ Proposal distribution: $y|x \sim N_{250}(0, 1.1^2 \cdot I)$



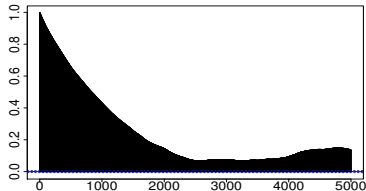
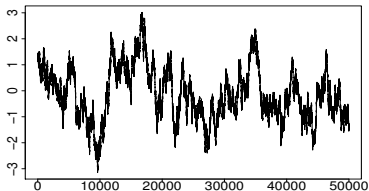
- ★ Trace plot of x^1



Toy example: Random walk proposals

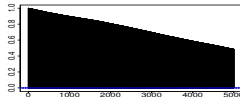
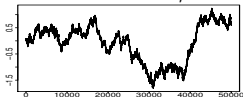
- ★ Target distribution: $x \sim N_{250}(0, I)$
- ★ Proposal distribution: $y|x \sim N_{250}(x, \sigma^2 \cdot I)$
- ★ Trace plot and acf of x^1

$\sigma = 0.05$, acceptance rate = 0.69

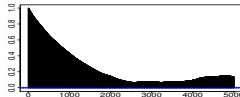
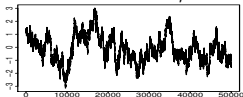


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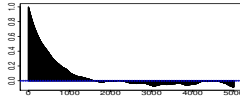
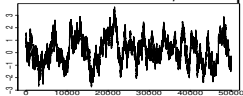
$\sigma = 0.01$, acceptance rate = 0.94



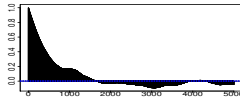
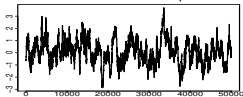
$\sigma = 0.05$, acceptance rate = 0.69



$\sigma = 0.10$, acceptance rate = 0.426



$\sigma = 0.20$, acceptance rate = 0.11



$\sigma = 0.30$, acceptance rate = 0.018

