Plug-in principle and bootstrapping

- ⋆ Assume: Have observed $x_1, \ldots, x_n$ iid from F
- ⋆ Empirical distribution $\widehat{F}$ puts probability $\frac{1}{n}$ on each $x_1, \ldots, x_n$
- ⋆ Parameter of interest: $\theta = t(F)$
- ⋆ Plug-in principle: $\widehat{\theta} = t(\widehat{F})$
- ⋆ Bootstrapping: Schematic view

Real world

$$\begin{tikzcd} F \arrow[r] & x = (x_1, \ldots, x_n) \arrow[d] \\ & \widehat{\theta} = s(x) \end{tikzcd}$$

$$\Longrightarrow$$

Bootstrap world

$$\begin{tikzcd} \widehat{F} \arrow[r] & x^\star = (x_1^\star, \ldots, x_n^\star) \arrow[d] \\ & \widehat{\theta}^\star = s(x^\star) \end{tikzcd}$$

Bootstrapping of standard error

- ⋆ Have $F \longrightarrow x = (x_1, \ldots, x_n)$
- ⋆ Empirical distribution $\widehat{F}$
- ⋆ Parameter of interest: $\theta = t(F)$
- ⋆ Estimator: $\widehat{\theta} = s(x)$
- ⋆ Want to estimate $\mathrm{SD}_F\left[\widehat{\theta}\right]$
- ⋆ Ideal bootstrap estimator: $\mathrm{SD}_{\widehat{F}}\left[\widehat{\theta}^\star\right]$
- ⋆ Pseudo code for bootstrap estimator $\widehat{\mathrm{SE}}_B$

  for $b = 1, 2, \ldots, B$ **do**
      Generate $x^{\star b} = (x_1^{\star b}, \ldots, x_n^{\star b})$
      Evaluate $\widehat{\theta}^\star(b) = s\left(x^{\star b}\right)$
  end for
  Estimate $\mathrm{SD}_{\widehat{F}}\left[\widehat{\theta}^\star\right]$ by

$$\widehat{\mathrm{SE}}_B = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\widehat{\theta}^\star(b) - \widehat{\theta}^\star(\cdot)\right)^2}$$