Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4305 Partial Differential Equations Spring 2009

Solutions for Problem Set Week 4

The exercises are from McOwen's book: *Partial differential equations.*

1 *Exercise 1.1.4(a)* We solve

$$xu_x + u_y = y, \qquad u(x,0) = x^2,$$

by the method of characteristics. The ODEs for the characteristics are:

$$\begin{aligned} \frac{dx}{dt} &= x, & x(0) &= s, \\ \frac{dy}{dt} &= 1, & y(0) &= 0, \\ \frac{dz}{dt} &= y, & z(0) &= s^2. \end{aligned}$$

We solve these, obtaining

$$x = se^t, \qquad y = t, \qquad z = \frac{1}{2}t^2 + s^2,$$

which gives $s = xe^{-y}$, hence

$$\underline{\underline{u(x,y) = z = \frac{1}{2}y^2 + x^2e^{-2y},}}$$

for all x, y.

2 *Exercise 1.1.6(a)* We solve

$$u_x + u^2u_y = 1, \qquad u(x,0) = 1,$$

by the method of characteristics. The ODEs for the characteristics are:

$$\begin{aligned} \frac{dx}{dt} &= 1\ , & x(0) &= s, \\ \frac{dy}{dt} &= z^2, & y(0) &= 0, \\ \frac{dz}{dt} &= 1\ , & z(0) &= 1. \end{aligned}$$

We solve these (solve first for x and z, then for y), obtaining

$$x = s + t, \qquad y = \frac{1}{3}(t+1)^3 - \frac{1}{3}, \qquad z = t + 1,$$

which gives $t + 1 = (3y+1)^{1/3}$, hence

$$\underline{\underline{u(x,y) = z = (3y+1)^{1/3}}}.$$

Thus, u is defined and continuous for all $y \in \mathbb{R}$, but it is not differentiable at $y = -1/3$ (since the third root function $t^{1/3}$ is defined and continuous for all $t \in \mathbb{R}$, but is not differentiable at the origin).