

$$V = \{f: \Omega \rightarrow \mathbb{R}\}$$

$(V, +, \cdot)$  VECTOR SPACE

$$L^p(\Omega) = \{f: \Omega \rightarrow \mathbb{R} \mid \int_{\Omega} |f|^p d^n \vec{x} < \infty\}, \quad 1 \leq p < \infty$$

$L^p(\Omega)$ ... SUBSPACE OF  $V$

$(L^p(\Omega), +, \cdot)$ ... VECTOR SPACE

$$\|f\|_p = \left( \int_{\Omega} |f|^p d^n \vec{x} \right)^{1/p}$$

$(L^p(\Omega), +, \cdot, \|\cdot\|_p)$ ... NORMED VECTOR SPACE

$$f, g \in L^p(\Omega)$$

$$\|f+g\|_p \leq \|f\|_p + \|g\|_p$$

$$\|f\|_p = 0$$

$$\Leftrightarrow f = 0 \text{ A.E.}$$

EQUIVALENCE CLASSES

$$p=2$$

$$\langle f, g \rangle = \int_{\Omega} f g d^n \vec{x} \quad (f, g \in L^2(\Omega))$$

INNER PRODUCT

$(L^2(\Omega), +, \cdot, \langle \cdot, \cdot \rangle)$ ... INNER PRODUCT SPACE

$$\|f\|_2 = \sqrt{\langle f, f \rangle}$$

$$L^\infty(\Omega) = \{f: \Omega \rightarrow \mathbb{R} \text{ MEASURABLE} \mid \text{ess-sup}(|f|) < \infty\}$$

$$\|f\|_\infty = \text{ess-sup}(|f|)$$