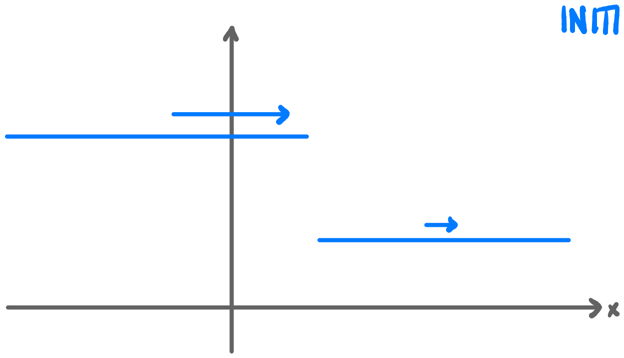
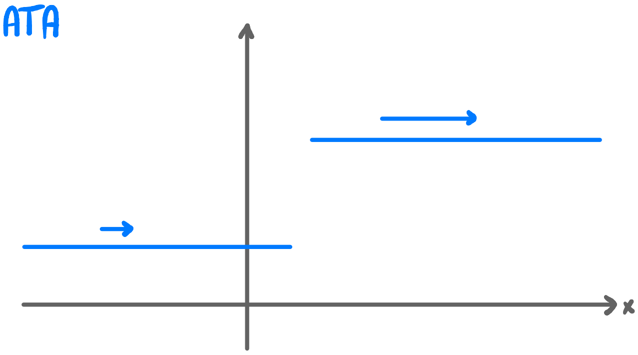
$u_t + uu_x = 0$

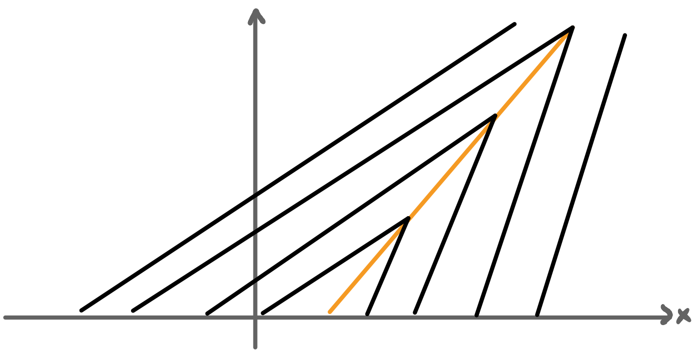
INITIAL DATA

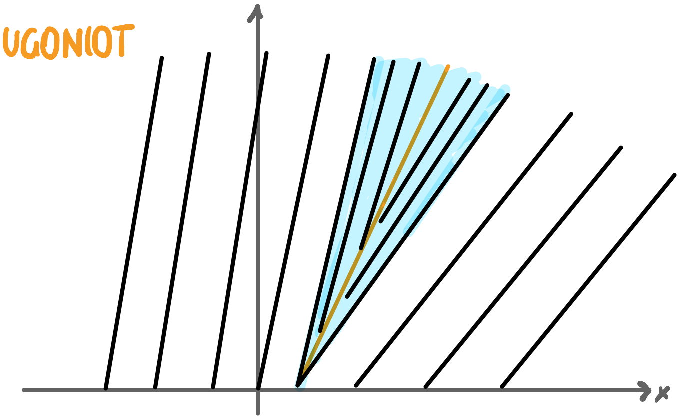




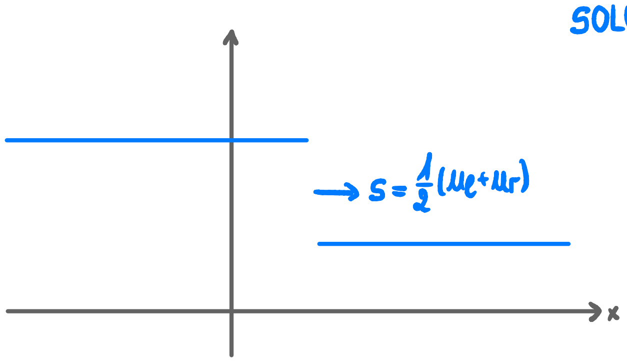
CHARACTERISTICS

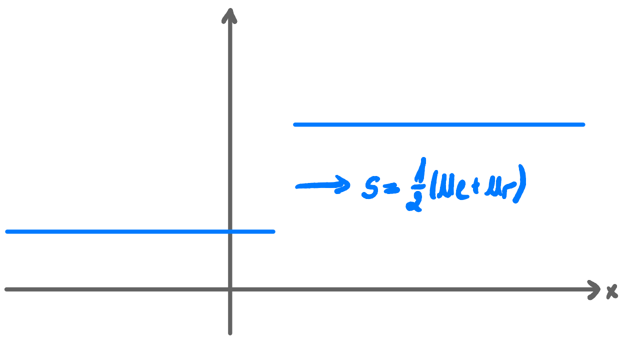
RANKINE-HUGONIOT





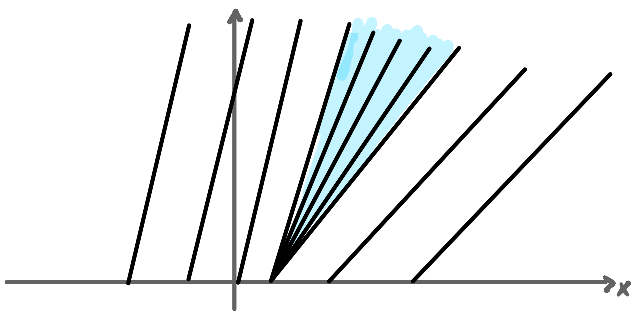
SOLUTION

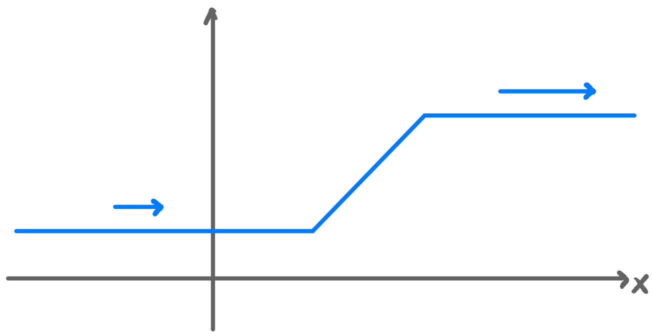




OTHER POSSIBILITY: RAREFACTION WAVE

CHARACTERISTICS





$$H^m(\Omega)=\{u\in L^2(\Omega)\mid D^\alpha u\in L^2(\Omega)\ \forall\, \|\alpha\|\le m\},\quad m\in\mathbb{N}_0$$

$$\Big\downarrow\ \langle u,v\rangle_{H^m}=\sum_{\|\alpha\|\le m}\langle D^\alpha u, D^\alpha v\rangle\quad \forall\, u,v\in H^m(\Omega)$$

L^2... INNER PRODUCT

$H^m(\Omega)$... INNER PRODUCT SPACE

$$\Big\downarrow\ \|u\|_{H^m}=\sqrt{\langle u,u\rangle_{H^m}}=\Big(\sum \|D^\alpha u\|_2^2\Big)^{1/2}\quad \forall\, u\in H^m(\Omega)$$

$H^m(\Omega)$... NORMED VECTOR SPACE

$\Big\downarrow$ $\{\psi_k\}\subseteq H^m(\Omega)$... CAUCHY SEQUENCE $\Rightarrow \{\psi_k\}$ CONVERGENT

$H^m(\Omega)$... HILBERT SPACE

$m=0$:

$H^0(\Omega)=L^2(\Omega)$

$\Big\downarrow$

$\mathcal{C}_c^\infty(\Omega)\subseteq L^2(\Omega)$
DENSE

$m=1$:

$H^1(\Omega)$

$\Big\downarrow$

$\mathcal{C}_c^\infty(\Omega)\subseteq H^1(\Omega)$
NOT DENSE

$\Big\downarrow$

$$H_0^1(\Omega)=\{u\in H^1(\Omega)\mid \exists\{\psi_k\}\subseteq \mathcal{C}_c^\infty(\Omega) \text{ SUCH THAT } \lim_{k\to\infty}\|\psi_k-u\|_{H^1}=0\}$$

$u\in H_0^1((a,b))$:

$u\in\mathcal{C}([a,b])$
$u(a)=0=u(b)$

$\Omega\subseteq\tilde{\Omega},\ u\in H_0^1((\Omega))$:

$$\tilde{u}(\vec{x})=\begin{cases} u(\vec{x}), & \vec{x}\in\Omega\\ 0, & \vec{x}\in\tilde{\Omega}\setminus\Omega\end{cases}\in H_0^1(\tilde{\Omega})$$