$$u_{tt} - \Delta u = 0 \quad \text{IN } (0,\infty) \times \Omega$$
$$u(0,\vec{x}) = g(\vec{x}),\ u_t(0,\vec{x}) = h(\vec{x}) \quad \forall \vec{x} \in \Omega$$

$$\Big\downarrow \ u \text{ CLASSICAL SOL}$$

$$\int_0^\infty \int_\Omega (u_{tt} - \Delta u)\psi(t,\vec{x})\, d^n\vec{x}\, dt = 0 \quad \forall \psi \in \mathcal{C}_c^\infty([0,\infty) \times \Omega)$$

$$\Big\downarrow$$

$$\int_0^\infty \int_\Omega (u\psi_{tt} + \langle \nabla u, \nabla \psi \rangle)(t,\vec{x})\, d^n\vec{x}\, dt = \int_\Omega h(\vec{x})\psi(0,\vec{x})\, d^n\vec{x} - \int_\Omega g(x)\psi_t(0,\vec{x})\, d^n\vec{x}$$

$$\forall \psi \in \mathcal{C}_c^\infty([0,\infty) \times \Omega)$$

$$\Big\downarrow \ \begin{array}{l} g, h \in L^1_{loc}(\Omega) \\ u(t,\cdot) \in H^1_0(\Omega) \quad \forall t \geq 0 \end{array}$$

u IS A WEAK SOLUTION IF

$$\int_0^\infty \int_\Omega (u\psi_{tt} + \langle \nabla u, \nabla \psi \rangle)(t,\vec{x})\, d^n\vec{x}\, dt = \int_\Omega h(\vec{x})\psi(0,\vec{x})\, d^n\vec{x} - \int_\Omega g(x)\psi_t(0,\vec{x})\, d^n\vec{x}$$

$$\forall \psi \in \mathcal{C}_c^\infty([0,\infty) \times \Omega)$$