$$u_t + a(u)u_x = 0$$
$$u(0,x) = g(x) \in C^1(\mathbb{R})$$

$\downarrow$ $(t,x(t))$ ... CURVE IN $\mathbb{R}^2$, $z(t) = u(t,x(t))$

$$z'(t) = u_t(t,x(t)) + u_x(t,x(t))x'(t)$$
$$z(0) = u(0,x(0)) = g(x(0))$$

$\downarrow$ $x'(t) = a(u(t,x(t))) = a(z(t))$ ... CHARACTERISTIC

$$x'(t) = a(z(t))$$
$$z'(t) = 0$$
$$z(0) = g(x(0)),\ x(0) \in \mathbb{R}$$

$\downarrow$ $a \in C^1(\mathbb{R})$

UNIQUE (LOCAL) SOLUTION $(x(t), z(t))$

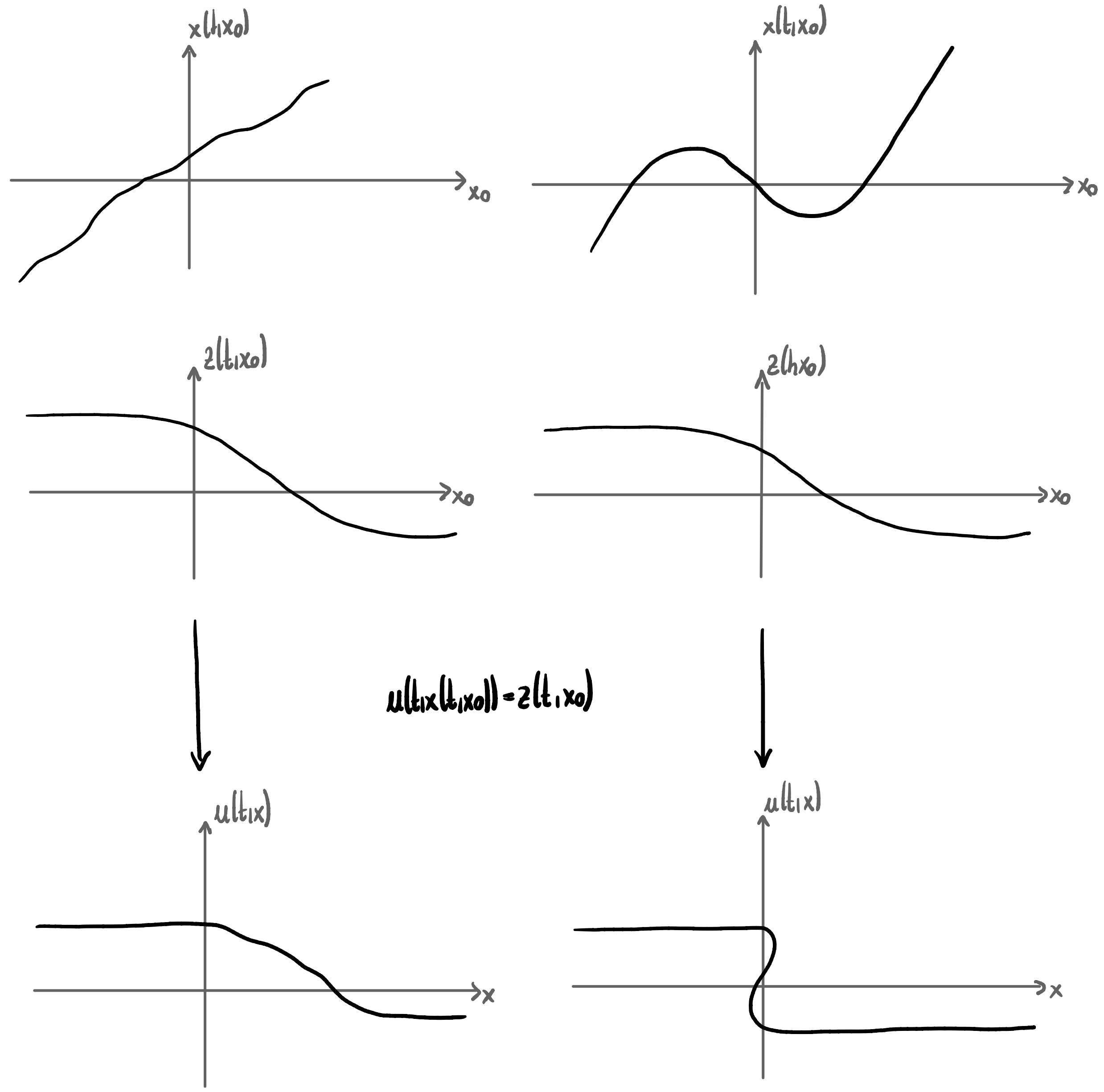
$\downarrow$

$u(t,x(t)) = z(t)$ ... SOLUTION ALONG $(t,x(t))$

$\downarrow$ $x(t,x_0)$ ... CHARACTERISTIC $x(t)$ WITH $x(0) = x_0$, $z(t,x_0)$ ... $z(t)$ WITH $z(0) = u(0,x_0)$

$t \in \mathbb{R}$

$x(t,\cdot): \mathbb{R} \to \mathbb{R}$, $x_0 \mapsto x(t,x_0)$ INVERTIBLE	$x(t,\cdot): \mathbb{R} \to \mathbb{R}$, $x_0 \mapsto x(t,x_0)$ NOT INVERTIBLE
$\downarrow$	$\downarrow$
$u(t,\cdot)$ WELL-DEFINED	$u(t,\cdot)$ NOT WELL-DEFINED


x(t,x0)
x0
x(t,x0)
x0
z(t,x0)
x0
z(t,x0)
x0
u(t,x(t,x0)) = z(t,x0)
u(t,x)
x
u(t,x)
x

$au_x + bu_y = c$

$u = g$ ON γ .... SMOOTH CURVE IN $\mathbb{R}^2$

(a, b, c ... FUNCTIONS OF (x, y, u))

$$\Big\downarrow \quad \begin{array}{l} (x(\tau), y(\tau)) \text{ ... CURVE IN } \mathbb{R}^2 \\ z(\tau) = u(x(\tau), y(\tau)) \end{array}$$

$$z'(\tau) = u_x(x(\tau), y(\tau))\, x'(\tau) + u_y(x(\tau), y(\tau))\, y'(\tau)$$

$$z(0) = u(x(0), y(0)) = g(x(0), y(0))$$

$$\Big\downarrow \quad \begin{array}{l} x'(\tau) = a(x(\tau), y(\tau), z(\tau)) \\ y'(\tau) = b(x(\tau), y(\tau), z(\tau)) \end{array}$$

$$x'(\tau) = a(x(\tau), y(\tau), z(\tau))$$
$$y'(\tau) = b(x(\tau), y(\tau), z(\tau))$$
$$z'(\tau) = c(x(\tau), y(\tau), z(\tau))$$
$$z(0) = g(x(0), y(0)), \quad (x(0), y(0)) \in \gamma$$

$$\Big\downarrow \quad a, b, c \in C^1(\mathbb{R})$$

UNIQUE (LOCAL) SOLUTION

$(x(\tau), y(\tau), z(\tau))$

$$\Big\downarrow$$

$u(x(\tau), y(\tau)) = z(\tau)$ ... SOLUTION ALONG $(x(\tau), y(\tau))$