$$
\begin{tikzcd}
\mathcal{R}[v]=\dfrac{2\mathcal{E}[v]}{\|v\|_2^2} \quad (v\in H^1_0(\Omega)) \arrow[d, "\text{POINCARÉ INEQUALITY}"] & \\
\mathcal{R}[v]\geq \dfrac{2}{k^2}>0 \quad \forall v\in H^1_0(\Omega) \arrow[d, "\text{RELLICH'S THM}"] & \\
\begin{array}{c}\exists \phi_1\in H^1_0(\Omega) \text{ WITH } \|\phi_1\|_2=1 \text{ SUCH THAT} \\ \lambda_1=\mathcal{R}[\phi_1]\leq \mathcal{R}[v] \quad \forall v\in H^1_0(\Omega)\end{array} \arrow[d, "v\in H^1_0(\Omega)"] & \\
t\mapsto \mathcal{R}[\phi_1+tv] \ (t\in\mathbb{R}) \text{ - MINIMUM AT } t=0 \arrow[d] & \\
\displaystyle\int_\Omega \left(\langle\nabla\phi_1,\nabla v\rangle-\lambda_1\phi_1 v\right) d^n\vec{x}=0 \arrow[d, "v\in H^1_0(\Omega) \text{ ARBITRARY}"] & \\
\begin{array}{c}\phi_1 \text{ IS A WEAK SOLUTION TO} \\ -\Delta\phi_1=\lambda_1\phi_1\end{array} \arrow[d] \arrow[r, "-\Delta\phi_1=f", "f=\lambda_1\phi_1\in H^1_0(\Omega)"'] & \phi_1\in H^3_{loc}(\Omega) \arrow[d, "f\in H^3_{loc}(\Omega)"] \\
\phi_1\in H^m_{loc}(\Omega) \ \forall m\in\mathbb{N} \arrow[d, "H^m_{loc}(\Omega)\subseteq \mathcal{C}^k(\Omega) \quad (k+\frac{1}{2}n<m)"] & \phi_1\in H^5_{loc}(\Omega) \arrow[l, "\text{INDUCTION}"'] \\
\phi_1\in\mathcal{C}^\infty(\Omega) &
\end{tikzcd}
$$