$$f \in \mathcal{C}^2(\mathbb{R}^3)$$

$$\Big\downarrow \; \partial B(\vec{x},\rho) = \{ \vec{y} \in \mathbb{R}^3 \mid \|\vec{y}-\vec{x}\| = \rho \}$$

$$\tilde{f}(\vec{x},\rho) = \frac{1}{4\pi\rho} \int_{\partial B(\vec{x},\rho)} f(\vec{\omega})\, dS(\vec{\omega}) \longrightarrow \lim_{\rho\to 0} \frac{\tilde{f}(\vec{x},\rho)}{\rho} = f(\vec{x}) \longrightarrow \lim_{\rho\to 0} \tilde{f}(\vec{x},\rho) = 0$$

$$\Big\downarrow$$

$$\tilde{f}_\rho(\vec{x},\rho) = \frac{\tilde{f}(\vec{x},\rho)}{\rho} + \frac{1}{4\pi\rho} \int_{B(\vec{x},\rho)} \Delta f(\vec{z})\, d^3\vec{z} \longrightarrow \lim_{\rho\to 0} \tilde{f}_\rho(\vec{x},\rho) = f(\vec{x})$$

$$\Big\downarrow$$

$$\tilde{f}_{\rho\rho}(\vec{x},\rho) = \frac{1}{4\pi\rho} \int_{\partial B(\vec{x},\rho)} \Delta f(\vec{\omega})\, dS(\vec{\omega}) \longrightarrow \lim_{\rho\to 0} \tilde{f}_{\rho\rho}(\vec{x},\rho) = 0$$

$$\Big\downarrow$$

$$\tilde{f}_{\rho\rho} - \Delta_x \tilde{f} = 0 \quad \text{ON} \quad \mathbb{R}^3 \times (0,\infty)$$

$$\tilde{f}(\vec{x},0) = 0, \quad \tilde{f}_\rho(\vec{x},0) = f(\vec{x})$$