$$u_{tt} - \Delta u = 0 \quad \text{ON } (0,\infty)\times\mathbb{R}^3$$

$$u(0,\vec{x}) = g(\vec{x}),\ u_t(0,\vec{x}) = h(\vec{x})$$

$$(g\in C^3(\mathbb{R}^3),\ h\in C^2(\mathbb{R}^3))$$

$\Big\downarrow$ $u\in C^2([0,\infty)\times\mathbb{R}^3)$ ... CLASSICAL SOLUTION

$$\tilde{u}(t,\vec{x},\rho) = \frac{1}{4\pi\rho}\int_{\partial B(\vec{x},\rho)} u(t,\vec{\omega})\,dS(\vec{\omega})$$

$\Big\downarrow$ $\tilde{f}(\vec{x},\rho) = \frac{1}{4\pi\rho}\int_{\partial B(\vec{x},\rho)} f(\vec{\omega})\,dS(\vec{\omega})$ FOR $f\in C^2(\mathbb{R}^3)$

$$\tilde{u}_{tt}(t,\vec{x},\rho) - \tilde{u}_{\rho\rho}(t,\vec{x},\rho) = 0 \quad (t>0,\ \rho>0,\ \vec{x}\in\mathbb{R}^3)$$

$$\tilde{u}(0,\vec{x},\rho) = \tilde{g}(\vec{x},\rho),\ \tilde{u}_t(0,\vec{x},\rho) = \tilde{h}(\vec{x},\rho)$$

$\Big\downarrow$ $\tilde{v}(t,\vec{x},\rho) = \begin{cases} -\tilde{u}(t,\vec{x},-\rho) & \rho<0 \\ \tilde{u}(t,\vec{x},\rho) & \rho\geq 0 \end{cases}$

$$\tilde{v}_{tt} - \Delta\tilde{v}_{\rho\rho} = 0 \quad \text{ON } (0,\infty)\times\mathbb{R}$$

$\Big\downarrow$ D'ALEMBERT'S FORMULA

$$\tilde{v}(t,\vec{x},\rho) = \frac{1}{2}\left[\tilde{g}(\vec{x},t+\rho) - \tilde{g}(\vec{x},t-\rho)\right] + \frac{1}{2}\int_{t-\rho}^{t+\rho}\tilde{h}(\vec{x},\tau)\,d\tau$$

$\Big\downarrow$ $u(t,\vec{x}) = \lim_{\rho\to 0}\frac{\tilde{u}(t,\vec{x},\rho)}{\rho}$

$$u(t,x) = \tilde{g}_t(\vec{x},t) + \tilde{h}(\vec{x},t)$$

$\Big\downarrow$ $(t,x)\in\Gamma_+(\vec{x}_0) = \{(s,\vec{y})\in\mathbb{R}^+\times\mathbb{R}^3 \mid \|\vec{y}-\vec{x}_0\| = s\}$ ... FORWARD LIGHT CONE

$u(t,\vec{x})$ DEPENDS ON $g(\vec{x}_0)$ AND $h(\vec{x}_0)$

$u_{tt} - \Delta u = 0$ ON $(0,\infty)\times\mathbb{R}^2$

$u(0,\vec{x}) = g(\vec{x})$, $u_t(0,\vec{x}) = h(\vec{x})$

$(g\in C^3(\mathbb{R}^2),\ h\in C^2(\mathbb{R}^2))$

$\Big\downarrow$ $u\in C^2([0,\infty)\times\mathbb{R}^2)$ ... CLASSICAL SOLUTION

$\hat{u}(t,\vec{x},z) = u(t,\vec{x})$ ... CLASSICAL SOLUTION TO

$\hat{u}_t - \Delta\hat{u} = 0$ ON $(0,\infty)\times\mathbb{R}^3$

$\hat{u}(0,\vec{x},z) = g(\vec{x})$, $\hat{u}_t(0,\vec{x},z) = h(\vec{x})$.

$\Big\downarrow$

$$u(t,\vec{x}) = \hat{u}(t,\vec{x},z) = \frac{\partial}{\partial t}\left(\frac{1}{4\pi t}\int_{\partial B((\vec{x},z),t)} g(\vec{y})\,dS(\vec{y},\vec{z})\right) + \frac{1}{4\pi t}\int_{\partial B((\vec{x},z),t)} h(\vec{y})\,dS(\vec{y},\vec{z})$$

$\Big\downarrow$ INTEGRAND INDEPENDENT OF $\vec{z}$

$$u(t,\vec{x}) = \frac{\partial}{\partial t}\left(\frac{t}{2\pi}\int_D \frac{g(\vec{x}+t\vec{y})}{\sqrt{1-\|\vec{y}\|^2}}\,d^2\vec{y}\right) + \frac{t}{2\pi}\int_D \frac{h(\vec{x}+t\vec{y})}{\sqrt{1-\|\vec{y}\|^2}}\,d^2\vec{y}$$

$\Big\downarrow$ $(t,\vec{x})\in\{(s,\vec{y})\in\mathbb{R}^+\times\mathbb{R}^2 \mid \|\vec{y}-\vec{x}_0\|\le s\}$

$u(t,\vec{x})$ DEPENDS ON $g(\vec{x}_0)$ AND $h(\vec{x}_0)$

$\Big\downarrow$ g,h ... COMPACT SUPPORT

$u(t,\cdot)$ COMPACT SUPPORT

$$u_{tt} - c^2 \Delta u = 0 \quad \text{ON } (0,\infty) \times \Omega$$
$$u(0,\vec{x}) = g(\vec{x}),\ u_t(0,\vec{x}) = h(\vec{x}) \quad \forall \vec{x} \in \Omega$$
$$u(t,\vec{x})\big|_{x \in \partial\Omega} = 0$$

$$\Big\Downarrow$$

Ω BOUNDED DOMAIN IN $\mathbb{R}^n$

u – CLASSICAL SOLUTION

$$\mathcal{E}[u](t) = \frac{1}{2}\int_\Omega (u_t)^2 + c^2 \|\nabla u\|^2)\, d^n\vec{x} < \infty$$

$$\Big\Downarrow$$

$u(t,\vec{x})\big|_{x \in \partial\Omega} = 0$

$$\mathcal{E}[u](t) = \mathcal{E}[u](0)$$

$$\Big\Downarrow$$

ASSUME: TWO SOLUTIONS u_1 & u_2 TO

$$v_{tt} - c^2 \Delta v = 0 \quad \text{ON } (0,\infty) \times \Omega.$$
$$v(0,\vec{x}) = g(\vec{x}),\ v_t(0,\vec{x}) = h(x) \quad \forall \vec{x} \in \Omega$$
$$v(t,\vec{x})\big|_{x \in \partial\Omega} = k(t,\vec{x})$$

$w = u_1 - u_2$ IS A SOLUTION TO

$$w_{tt} - c^2 \Delta w = 0 \quad \text{ON } (0,\infty) \times \Omega$$
$$w(0,\vec{x}) = 0 = w_t(0,\vec{x}) \quad \forall \vec{x} \in \Omega$$
$$w(t,\vec{x})\big|_{x \in \partial\Omega} = 0$$

$$\Big\Downarrow$$

$$\mathcal{E}[w](0) = 0$$

$$\Big\Downarrow$$

$$\mathcal{E}[w](t) = \frac{1}{2}\int_\Omega \underset{\geq 0}{(w_t)^2} + c^2 \underbrace{\|\nabla w\|^2}_{\geq 0}\, d^n\vec{x} = 0$$

$$\Big\Downarrow$$

$$w \equiv 0 \iff u_1 \equiv u_2$$