$\tilde{H}(t,x) = \frac{1}{\sqrt{4\pi t}}\, e^{-\frac{x^2}{4t}}$ ... HEAT KERNEL IN 1D

$\tilde{H}(t,x) \in C^\infty((0,\infty)\times\mathbb{R})$

$\tilde{H}_t - \tilde{H}_{xx} = 0$ ON $(0,\infty)\times\mathbb{R}$

$\int_{\mathbb{R}} \tilde{H}(t,z)\,dz = 1$

$\Big\downarrow$ $\varphi(x) \in C^0(\mathbb{R})$ BOUNDED

$u(t,x) = \int_{\mathbb{R}} \varphi(z)\,\tilde{H}(t,x-z)\,dz \in C^\infty((0,\infty)\times\mathbb{R})$

$\Big\downarrow$

$u_t - u_{xx} = 0$

$u(0,x) = \varphi(x)$

$H(t,\vec{x}) = \frac{1}{(\sqrt{4\pi t})^n}\, e^{-\frac{\|\vec{x}\|^2}{4t}} = \tilde{H}(t,x_1)\tilde{H}(t,x_2)\dots\tilde{H}(t,x_n)$ ... HEAT KERNEL IN nD

PROPERTIES OF $\tilde{H}$ IMPLY

$H(t,\vec{x}) \in C^\infty((0,\infty)\times\mathbb{R}^n)$

$H_t - \Delta H = 0$

$\int_{\mathbb{R}^n} H(t,\vec{z})\,d^n\vec{z} = 1$

$\Big\downarrow$ $\varphi(x) \in C^0(\mathbb{R}^n)$ BOUNDED

$u(t,\vec{x}) = \int_{\mathbb{R}^n} \varphi(\vec{z})\,H(t,\vec{x}-\vec{z})\,d^n\vec{z} \in C^\infty((0,\infty)\times\mathbb{R}^n)$

$\Big\downarrow$

$u_t - \Delta u = 0$

$u(0,\vec{x}) = \varphi(\vec{x})$