$u_t - \Delta u = f$ ON $(0,\infty)\times\mathbb{R}^n$

$u(0,\vec{x}) = 0$

$(f \in C^2([0,\infty)\times\mathbb{R}^n)$

$$\Big\downarrow\; s>0$$

$\eta(t,\vec{x};s)$ SOLUTION TO

$\eta_t - \Delta\eta = 0$ ON $(s,\infty)\times\mathbb{R}^n$

$\eta(s,\vec{x};s) = f(s,\vec{x})$

$$\Big\downarrow\; H(t,\vec{x}) \text{ .. } n\text{ DIM HEAT KERNEL}$$

$$\eta(t,\vec{x};s) = \int_{\mathbb{R}^n} f(s,\vec{y})\,H(t-s,\vec{x}-\vec{y})\,d^n\vec{y}$$

$$\Big\downarrow$$

$$u(t,\vec{x}) = \int_0^t \eta(t,\vec{x};s)\,ds = \int_0^T\int_{\mathbb{R}^n} f(s,\vec{y})\,H(t-s,\vec{x}-\vec{y})\,d^n\vec{y}\,ds$$

$$\Big\downarrow\; v \text{ SOLUTION TO } \begin{array}{l} v_t - \Delta v = 0 \text{ ON } (0,\infty)\times\mathbb{R}^n \\ v(0,\vec{x}) = g(\vec{x}) \end{array}$$

$w = u + v$ SOLUTION TO

$w_t - \Delta w = f$

$w(0,\vec{x}) = g(\vec{x})$

$u_t - \Delta u = 0$ ON $\Omega_T = (0,T) \times \Omega$

$u|_{\Gamma_T} = g \qquad \Gamma_T = (\{0\} \times \bar{\Omega}) \cup ((0,T) \times \partial\Omega)$

(Ω DOMAIN)

$$
\begin{tikzcd}[column sep=small]
 & u \in C(\bar{\Omega}_T) \cap C^2(\Omega_T) \arrow[dl, "u_t - \Delta u \leq 0"'] \arrow[dd, "u_t - \Delta u = 0"] \arrow[dr, "u_t - \Delta u \geq 0"] & \\
u(t,x) \leq \max\limits_{\Gamma_T} u \arrow[dr] & & \min\limits_{\Gamma_T} u \leq u(t,x) \arrow[dl] \\
 & \min\limits_{\Gamma_T} u \leq u(t,x) \leq \max\limits_{\Gamma_T} u &
\end{tikzcd}
$$