$$u_t-\Delta u=0 \quad \text{ON } \Omega_T=(0,T)\times\Omega$$
$$u|_{\Gamma_T}=g \quad \Gamma_T=(\{0\}\times\bar\Omega)\cup((0,T)\times\partial\Omega)$$
(Ω DOMAIN)

$$\begin{tikzcd}
 & u\in C(\bar\Omega_T)\cap C^2(\Omega_T) \arrow[dl, "u_t-\Delta u\le 0"'] \arrow[dd, "u_t-\Delta u=0"] \arrow[dr, "u_t-\Delta u\ge 0"] & \\
u(t,x)\le\max\limits_{\Gamma_T}u \arrow[dr] & & \min\limits_{\Gamma_T}u\le u(t,x) \arrow[dl] \\
 & \min\limits_{\Gamma_T}u\le u(t,x)\le\max\limits_{\Gamma_T}u \arrow[dl, "u|_{\Gamma_T}=0"'] \arrow[dr, "u|_{\Gamma_T}=g"] & \\
u(t,\vec x)\equiv 0 \arrow[d, "\substack{u_t-\Delta u=f\\ u|_{\Gamma_T}=g}"'] & & -\max\limits_{\Gamma_T}|g|\le u(t,\vec x)\le\max\limits_{\Gamma_T}|g| \arrow[d, "\substack{(u_i)_t-\Delta u_i=f\\ u_i|_{\Gamma_T}=g_i}\quad (i=1,2)"] \\
\exists!\ \text{SOLUTION} & & |u_1-u_2|(t,\vec x)\le\max\limits_{\Gamma_T}|g_1-g_2|
\end{tikzcd}$$

$$u\in \mathcal{C}([0,T]\times\mathbb{R}^n)\cap \mathcal{C}^2((0,T)\times\mathbb{R}^n)$$

$$u(t,\vec{x})\le Ae^{a\|\vec{x}\|^2}$$

$$\Big\downarrow \quad v(t,\vec{x})=u(t,\vec{x})-\varepsilon\underbrace{\frac{1}{(T+\delta-t)^{n/2}}e^{\frac{\|\vec{x}\|^2}{4(T+\delta-t)}}}_{B(T+\delta-t,\vec{x})}$$

$$v(0,\vec{x})\le u(0,\vec{x}) \quad \forall \vec{x}\in\mathbb{R}^n$$

$$v(t,\vec{x})\le \left[Ae^{\left(a-\frac{1}{4(T+\delta)}\right)R^2}-\frac{\varepsilon}{(T+\delta)^{n/2}}\right]e^{\frac{R^2}{4(T+\delta)}} \quad \forall \|\vec{x}\|=R,\ 0\le t\le T$$

$$\Big\downarrow \quad u(0,\vec{x})\le M$$

$\exists \hat{R}>0$ SUCH THAT $\forall R>\hat{R}$

$$\max_{\Gamma_{T,R}} v(t,\vec{x})\le M$$

$$\left[\Gamma_{T,R}=(\{0\}\times B(\vec{0},R))\cup((0,T)\times\partial B(0,R))\right]$$

$$\Big\downarrow \quad \text{MAXIMUM PRINCIPLE} \quad \Omega_{T,R}=(0,T)\times B(0,R)$$

$$\max_{\bar{\Omega}_{T,R}} v(t,\vec{x})\le M \quad \forall R>\hat{R}$$

$$\Big\downarrow \quad \varepsilon \text{ ARBITRARY}$$

$$\max_{\bar{\Omega}_{T,R}} u(t,\vec{x})\le M \quad \forall R>\hat{R}$$

$$\Big\downarrow \quad u\in\mathcal{C}([0,T]\times\mathbb{R}^n)$$

$$u(t,\vec{x})\le M \quad \forall (t,\vec{x})\in[0,T]\times\mathbb{R}^n$$