M4: Count and continuous positive response (w2)

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Gamma distribution

$$f(y_i) = \frac{1}{\Gamma(\nu)} \left(\frac{\nu}{\mu_i}\right)^{\nu} y_i^{\nu-1} e^{-\frac{\nu}{\mu_i} y_i} \quad , \quad y_i > 0$$

$\frac{\nu}{\mu_i}$

$$= \exp\left(\frac{-\frac{1}{\mu_i} \cdot y_i - \left(-\ln \frac{1}{\mu_i}\right)}{\frac{1}{\nu}} + c(y_i, \nu) \right)$$

Class notes M1

$$\theta_i = \frac{1}{\mu_i} \quad , \quad \phi = \frac{1}{\nu}$$

$$E(Y_i) = \mu_i \quad , \quad Var(Y_i) = \frac{\mu_i^2}{\nu}$$

Canonical link: $\eta_i = -\frac{1}{\mu_i}$

seldom used because: $\mu_i > 0 \Rightarrow \eta_i < 0$

we want $(-\infty, \infty)$

Competitor: $\eta_i = \ln(\mu_i)$

$\mu_i > 0 \rightarrow \eta_i \in (-\infty, \infty)$

Gamma GLM

1. $Y_i \sim Ga(\mu_i, \nu)$
2. $\eta_i = x_i^T \beta$
3. $\eta_i = \ln(\mu_i)$ ← most popular

R-printout from glm - family = Gamma

We now estimate dispersion parameter, $\phi = \frac{1}{\nu}$,
by ML $\hat{\phi} = 0.02265 = \frac{1}{\hat{\nu}}$

So that $Var(Y_i) = \frac{\mu_i^2}{\nu} = \mu_i^2 \cdot \phi$ ← scaling the variance

⇓

use t, not z in print-out

The Wald test in column 4 assumes a t-distribution with $df = n - p$ instead of asymptotic N.

This is since the dispersion parameter is estimated.

Deviance: $\left.\begin{array}{l}\text{scaled} \leftarrow \text{likelihood} \\ \text{unscaled} \leftarrow \text{not including } \phi\end{array}\right\}$ IL 11.10

NB NB

bin/Poiss: $\phi = 1$ not an issue there

Compare models with AIC (choose min AIC)