

MT7: Generalized linear mixed models - GLMM

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Aim: analyse correlated observations with non-normal response.

GLM: (Y_i, x_i) $i=1, \dots, n$ independent pairs

1. Random component: exponential family

Beeches: $Y_i \sim \text{Poisson}(\lambda_i)$, $\mu_i = E(Y_i) = \lambda_i$

2. Systematic component: linear predictor

$$\eta_i = x_i^T \beta$$

3. Link/response function

$$\eta_i = g(\mu_i) \quad \text{or} \quad \mu_i = h(\eta_i)$$

Beeches: $\eta_i = \ln(\mu_i) = \ln(\lambda_i)$, $\lambda_i = \exp(\eta_i)$

Beeches

i) fitall: block like $\hat{\mu}_i = \exp(\hat{\eta}_i) = \exp(\hat{\beta}_0 + \hat{\beta}_1 x_{ij})$

ii) fitbeach: many $\hat{\mu}_{ij} = \exp(\hat{\eta}_{ij}) = \exp(\hat{\beta}_0 + \beta_{beach} i + \hat{\beta}_1 x_{ij})$

NAP_{ij}



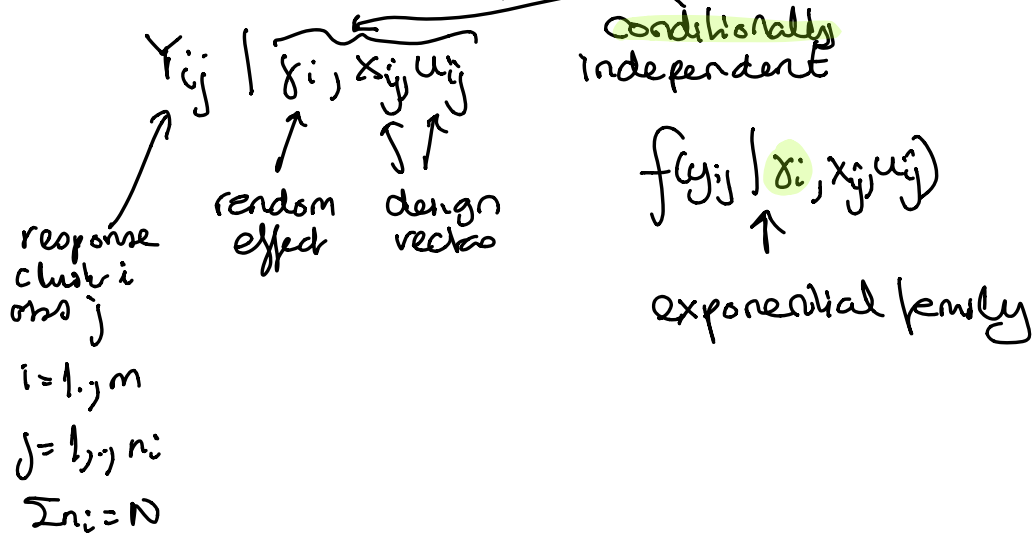
GLMM: we add random component to linear predictor AND

let $y_i \sim N(0, \sigma^2)$ as for lower

+ distribution of Y_{ij} is conditional on y_i

GLMM:

Distributional assumptions



Structural assumptions

$\mu_{ij} = E(Y_{ij} \mid \delta_i, x_i, u_i)$ conditional mean

$\eta_{ij} = \text{linear predictor} = x_{ij}^T \beta + u_{ij}^T \gamma_i$

$\eta_{ij} = g(\mu_{ij})$ link function

$\mu_{ij} = h(\eta_{ij})$ response function

Distributional assumption for random effects

γ_i i.i.d. $i=1, \dots, m$

$\gamma_i \sim N(0, Q)$
 \uparrow $(q+1) \times (q+1)$ positive definite

How to simulate data: [not in lecture]

- decide on β and elements in Q
- draw $\gamma_i \sim N(0, Q)$
- $x_{ij}, u_{ij} \leftarrow$ draw or fixed
- $\eta_{ij} = x_{ij}^T \beta + u_{ij}^T \gamma_i$ and $\mu_{ij} = h(\eta_{ij})$
- draw $y_{ij} \sim f(\mu_{ij})$
 \uparrow Bernoulli: $\text{Poisson}(\exp(x_{ij}^T \beta + u_{ij}^T \gamma_i))$

GLMM with random intercept

$$\eta_{ij} = x_{ij}^T \beta + \gamma_{0i}$$

\uparrow
 $N(0, \tau_0^2)$

Beaches:

- ML & Laplace ^{NAP} ↓

$$\hat{\mu}_{ij} = \exp(\hat{\beta}_0 + \hat{\beta}_1 x_{ij} + \gamma_{0i})$$
$$= \exp(\hat{\beta}_0 + \hat{\beta}_1 x_{ij}) \cdot \underbrace{\exp(\gamma_{0i})}$$

Not just a shift in
intercept on exp scale:

but a mult. effect!

→ remember LMM: $\mu_{ij} = \hat{\beta}_0 + \hat{\beta}_1 x_{ij} + \gamma_{0i}$

Conditional model:

$$f(y_{ij} | x_i) \quad \text{and} \quad f(x_i) \leftarrow \text{normal}$$

\uparrow Poisson
binomial
gamma
normal

Joint model: $f(y_{ij}, x_i) = f(y_{ij} | x_i) \cdot f(x_i)$

of marginal model:

$$f(y_{ij}) = \int_{\delta_i} f(y_{ij} | \delta_i) f(\delta_i) d\delta_i$$

\uparrow
N
 \uparrow
?

\uparrow \uparrow
N
Poisson

\uparrow \uparrow
N

← closed form expression
only in very special cases!

Parameter estimation:

\uparrow
ML, but
performed
numerically

parameter estimation
becomes
challenging

\Downarrow
Compulsory SE