M8: Summing-up

20.11.2017

1) Models:

Exponential family: (univariate) Y_i

$$f(y_i) = \exp\left(\frac{y_i\theta_i - b(\theta_i)}{\phi}w_i + c(y_i, w_i, \phi)\right)$$

$$E(Y_i) = b'(\theta_i), \quad \mathrm{Var}(Y_i) = b''(\theta_i)\cdot\frac{\phi}{w_i}$$

MLR $\varepsilon_i \sim N(0, \tau^2)$
$Y_i = x_i^T\beta + \varepsilon_i$

$Y = X\beta + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2 I)$

$Y_i \sim N(\mu_i, \sigma^2)$

$i=1,\dots,n, \quad \mu_i = X_i^T\beta$

independent Y_i's

MLR (prepared for GLM)

1) $Y_i \sim N$
   $\mu_i = E(Y_i)$
   $\mathrm{Var}(Y_i)$

2) $\eta_i = x_i^T\beta$

3) $\eta_i = \mu_i$

GLM

1) $Y_i \sim f$ exp. fam
   $E(Y_i) = \mu_i$
   $\mathrm{Var}(Y_i) = \sigma_i^2$

2) $\eta_i = x_i^T\beta$

3) $\eta_i = g(\mu_i), \mu_i = h(\eta_i)$

$\theta_i \longrightarrow \mu_i \longrightarrow \eta_i \longrightarrow \beta$

~~GLM~~
Not correct

~~$Y_i = x_i^T\beta + \varepsilon_i \leftarrow \varepsilon_i \sim \text{bin}$~~
~~$\varepsilon_i \sim \text{Poisson}\dots$~~

Correlated Y_{ij}'s $Y_i = \begin{bmatrix} Y_{i1} \\ \vdots \\ Y_{in_i} \end{bmatrix}$ **LMM**

$Y_i = X_i\beta + U_i\gamma_i + e_i$
         ↑
      random
      effect

GLMM

2) $\eta_i = x_i^T\beta + \boxed{u_i^T\gamma_i}$

1) $Y_i | \gamma_i$

2) $\eta_i = x_i^T\beta + u_i^T\gamma_i$

1) $Y_i | \gamma_i$

2) Likelihood: shown on "OneNote overview"

3) Inference: —''— ← Wald, LRT

4) Important: $\hat{\beta} \simeq N(\beta, F^{-1}(\hat{\beta}))$ ← in general $n \to \infty$

                  ↑ expected Fisher info

   LRT & Wald $\simeq \chi^2$ ← asymptotic

5) Adequacy: focus on deviance test (remember H_0: model is good)
   and AIC

Then → print-out from R : separate file
Then → exam : separate file

One-page overview

1 Exponential family

2 MLR → 3 MLR prepared for GLM

4 GLM model → 5 GLM likelihood inference

→ 6 Fischer scoring → 7 Requirements for GLM

8 Further GLM inference → 9 Deviance → 10 AIC GLM

11 LMM → 12 LMM random intercept & slope

13 Model selection for LMM

14 GLMM

15 Binary regression
16 Poisson regression → 17 Gamma regression

Not important, compared to previous ones