Eksamen TMA4135 kont. '19

Oppgave 8

a) Gitt initialverdiproblemet

$$y' = \sqrt{y}, \quad y(0) = 1.$$

Skriv ned en *fullstending* algoritme for å finne en tilnærmelse til $y(2)$ ved bruk av implisitt (baklengs) Eulers metode, med steglengde $h = 2/N$.

Utfør et steg med algoritmen med $h = 0.1$, dvs. finn en tilnærmelse til $y(0.1)$.

NB! Algoritmen må gjerne skrives i form av kode i f.eks. MATLAB eller Python. Den skal være tilstrekkelig detaljert til at den kan implementeres.

Eksamen TMA4125 vår '22

Problem 3. (Ralston's method, 14 points)

For the ordinary differential equation

$$y'(t) = -6y(t), \quad \text{with} \quad y(0) = 1,$$

consider *Ralston's method* given by the following Butcher tableau:

0	0	0
2/3	2/3	0
	1/4	3/4

Using the tableau and expanding the stage derivatives k_i, we can write the solution y_{n+1} in terms of the previous one, y_n, and of the time-step size $h > 0$. More precisely:

$$y_{n+1} = R(h)\, y_n, \quad \text{so that} \quad y_n = [R(h)]^n\, y(0),$$

in which $R(h)$ is a second-degree polynomial.

a) How many stages does this Runge–Kutta method have?

b) Determine the polynomial $R(h)$.

Problem 3 **Numerical methods for ODEs [10 pts]** Consider python code that solves an initial value problem with a Runge-Kutta method and prints the resulting x and y value after N steps:

```
def f(x, y):
  return x * y**2

def step(x, y, h):
  k1 = f(x, y)
  k2 = f(x + h, y + h * k1)
  y_new = y + (h / 2) * (k1 + k2)
  x_new=x+h
  return x_new, y_new

x = 0
y = 0.5
h = 0.1
N=20

for n in range(N):
  x,y=step(x,y,h)

print(x,y)
```

a) What is the initial value problem that the code solves?

b) Write the Butcher tableau for the method used. Is the method explicit or implicit? motivate your answer.