Julian $(\Bbbk = \overline{\Bbbk})$

1. Determine (Q, I) for $\Lambda =$

(a) upper triangular $n\times n$-matrices

(b) $\left\{ \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & a \end{pmatrix} \right\}$

(c) $\Bbbk\, \mathbb{Z}/p\mathbb{Z}$ (divide char p, other)

(d) $\Bbbk S_3$ (divide char 2, 3, other)

2. Show that $\Bbbk[x]/(x^2 - x^3)$ is fin. dim. but $(x^2 - x^3)$ is not admissible. Determine (Q, I).

3. (a) Show that $\Bbbk Q$ is hereditary, i.e. that submodules of projectives are projective. (Q acyclic)

(b) Deduce that ${}_{\Bbbk Q/I} L/IL$ is projective

(c) Show that $P_\bullet \longrightarrow \mathbb{L}$ given by $P_{2n} = I^n/I^{n+1}$, $P_{2n+1} = I^n J/I^{n+1} J$ is a proj. resolution

(d) Use tensor-hom-adjunction to show $\mathrm{Ext}^*(\mathbb{L}, \mathbb{L}) = \dots$