① Some authors define an A_∞-algebra via

$$\tilde{\mu}_n : A^{\otimes n} \longrightarrow A$$ satisfying

$$\sum (-1)^{rs+t}\, \tilde{\mu}_{r+1+t}\left(1^{\otimes r} \otimes \tilde{\mu}_s \otimes 1^{\otimes t}\right) = 0$$

Prove that the two conditions give rise to equivalent notions.

$$\left(\underline{\text{Hint}}: \left(\sigma^{\otimes n}\right)^{-1} = (-1)^{\frac{n(n+1)}{2}} \left(\sigma^{-1}\right)^{\otimes n}\right)$$

② An $\underline{A_\infty\text{-coalgebra}}$ is a graded vector space C together with maps $\Delta_n : C \longrightarrow C^{\otimes n}$ of degree $n-2$ satisfying

$$\sum (-1)^{rs+t} \left(1^{\otimes r} \otimes \Delta_s \otimes 1^{\otimes t}\right) \Delta_{r+1+t} = 0$$

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Prove that graded k-duality $\mathbb{D} = \mathrm{Hom}_k(-, k)$ applied to a finite dimensional A_∞-coalgebra gives rise to an A_∞-algebra with

$$\mu_n : (\mathbb{D}C)^{\otimes n} \longrightarrow \mathbb{D}C \text{ given by } \mu_n = (-1)^n \, \mathbb{D}(\Delta_n) \circ \iota_n$$

where $\iota_n : (\mathbb{D}C)^{\otimes n} \longrightarrow \mathbb{D}(C^{\otimes n})$

$\varphi_1 \otimes \dots \otimes \varphi_n \longmapsto \varphi_1 \otimes \dots \otimes \varphi_n$

(Koszul sign rule)