

Let  $A$  be an  $A_\infty$  category and

$$(M, \delta_m) \xleftarrow{\Psi_{m-1}} \dots \xleftarrow{\Psi_1} (M_1, \delta_1)$$

a sequence of morphisms in  $\text{Tw} A$

Show that if  $\deg \Psi_k = 1$  and  $M_{\text{Tw}}(\Psi_k) = 0$

then  $M_{\text{Tw}}(\Psi_{m-1}, \dots, \Psi_k, \dots, \Psi_1) = \pm$

$$M_{\text{Tw}}(\Psi_{m-1}, \tilde{\Psi}_{k+1}, \tilde{\Psi}_{k-1}, \Psi_1)$$

where

$$\tilde{\Psi}_{k+1} : \text{Cone}(\Psi_k) \rightarrow (M_{k+1}, \delta_{k+1}) = \begin{bmatrix} \Psi_{k+1} & \\ & \end{bmatrix}$$

$$\tilde{\Psi}_{k-1} : (M_{k-1}, \delta_{k-1}) \rightarrow \text{Cone}(\Psi_k) = \begin{bmatrix} & \\ & \Psi_{k-1} \end{bmatrix}$$

$$\text{Cone}(\Psi_k) = (M_k \oplus M_{k+1}, \begin{pmatrix} 0 & \delta \\ \Psi_k & 0 \end{pmatrix})$$