STOCHASTIC RESERVOIR CHARACTERIZATION CONDITIONED ON SEISMIC DATA

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Abstract. Well observations and seismic data are integrated in a consistent Bayesian framework for reservoir characterization. The uncertainty is represented by sampling from the posterior probability density function. The model uses well observations of porosity, permeability and seismic reflection coefficients and zero-offset or stacked seismic data. The seismic traces are modeled as reflection coefficients convolved with a seismic wavelet plus noise. A simple example demonstrates the technique.

1. Introduction

Reservoir characterization is an important task in the reservoir evaluation process. Stochastic modeling is in frequent use to integrate various types of information and to make quantification of uncertainty possible. Two types of information are available: *(i)* general reservoir knowledge with observations from geological analogues and *(ii)* reservoir specific observations such as well logs, seismic data and production history. Stochastic reservoir characterization in a Bayesian framework has proven useful for the integration of reservoir information and for assessing uncertainties, see Lia et al. (1996) and Omre and Tjelmeland (1996).

Integration of seismic data is particularly important since it provides good spatial coverage over the reservoir. Doyen (1988) used a standard cokriging model to integrate well observations and seismic data. Later, several authors extended this model to include an indicator formalism, see Araktingi and Bashore (1992), Doyen and Guidish (1992) and Zhu and Journel (1993), but these authors did not use an explicit model for the seismic reflections. Bortoli et al. (1993) and Haas and Dubrule (1994) used simple seismic modeling to constrain the stochastic reservoir model. Stochastic modeling is also in frequent use in the geophysical community, see Tarantola (1987). The emphasis has been on a Bayesian formalism and not so much on spatial aspects. Work along these lines is also presented in Brac et al. (1992).

In this paper seismic data are integrated into the reservoir characterization in a consistent Bayesian framework. It is shown that under a Gaussian based model, similar to the one implicitly used in Haas and Dubrule (1994), the problem can be solved analytically. This makes correct sampling and assessment of uncertainties possible, in contrast to previous work. Our model uses well observations of porosity, permeability and seismic reflection coefficients and zero-offset or stacked seismic data. The seismic traces are modeled as reflection coefficients convolved with a seismic wavelet plus noise. The shape of the seismic wavelet is characterized by a few parameters which are also estimated.

2. Stochastic Reservoir Characterization

The reservoir characteristics of interest, i.e. porosity, permeability, saturations etc., are represented by a multivariate, spatial variable r. The characteristics are largely unknown at the stage of evaluation. It is convenient to represent it by a stochastic variable R, with associated prior probability density function, pdf, $f(r)$. This prior pdf must be established by geoscientists based on general reservoir knowledge and observations from geological analogues. Often it is necessary to split the prior pdf according to $f(r) = \int f(r|\theta) f(\theta) d\theta$ in order to properly represent the prior uncertainty. The model parameters, like net-to-gross, expected values, variances etc, are defined to be stochastic variables Θ with prior pdf $f(\theta)$ which may be assessed as in an empirical Bayes approach. The reservoir characteristics given the model parameters have prior $f(r|\theta)$. This corresponds to the heterogeneity modeling frequently used in reservoir description.

At the evaluation stage, some reservoir specific observations are available, for example well observations, seismic data and production tests. Denote these observations o, and their stochastic counterpart O. The link between the observations and the reservoir characteristics can be modeled as $O = g(r, \psi) + U$, with $g(\cdot, \psi)$ being a transfer function depending on some parameters ψ representing the data acquisition procedure. This transfer function may be very complex as in geophysics and production tests. The observation error term U represents both modeling error and measurement errors. The link between reservoir characteristics and observations is represented by an observation likelihood function $f(o|r)$, interpreted as the probability for observing o given the reservoir r. If stochastically defined data acquisition parameters are included one gets $f(o|r) = \int f(o|r, \psi) f(\psi) d\psi$, with $f(\psi)$ being a prior pdf on these parameters provided by the geoscientists based on experience with the acquisition equipment.

The objective is to determine the posterior pdf $f(r|o)$ for the reservoir characteristics given the reservoir specific observations. It can be expressed as

$$\begin{aligned} f(r|o) &= \text{const} \times f(o|r)f(r) \\ &= \text{const} \times \int f(o|r,\psi)f(\psi)d\psi \int f(r|\theta)f(\theta)d\theta. \end{aligned} \tag{1}$$

For a few particular choices of pdf's this can be calculated analytically, for some choices of pdf's it can be sampled using current computer technology, while in the general case it is completely intractable. Omre and Tjelmeland (1996) give a more thorough discussion.

Whenever sampling based approaches are used, it is more convenient to consider

$$f(r,\theta,\psi|o) = \text{const} \times f(o|r,\psi)f(\psi)f(r|\theta)f(\theta). \tag{2}$$

By sampling the triplet (R, Θ, Ψ), samples of R are implicitly obtained.

3. Operational Model

There are two aspects of the model in the previous section that are crucial: *(i)* the reservoir characteristics, R, are multivariate and spatial, thus high dimensional, and *(ii)* the transfer functions of data acquisition, $g(\cdot,\cdot)$, often require extensive computer resources. The reservoir model parameters Θ and the data acquisition parameters Ψ can often be defined in reasonably low dimensions and are simpler to handle. In this section a simple model is defined that is suitable for data integration and also possible to sample. The developments are along the lines of the results in Tjelmeland (1996).

3.1. RESERVOIR CHARACTERISTICS

Let the reservoir characteristics be

$$R : \{R(x,t) = (\Phi(x,t), K(x,t), C(x,t))^T; x \in \mathcal{A}; t \in \mathcal{T}\}$$

with $\Phi(\cdot,\cdot)$, $K(\cdot,\cdot)$ and $C(\cdot,\cdot)$ being porosity, permeability and seismic reflection coefficients respectively; x being lateral coordinate in the domain of interest $\mathcal{A}$; and t being the seismic reflection time in the domain of interest $\mathcal{T}$. The seismic reflection coefficients are defined as the derivative of the logarithm of the seismic impedance.

3.2. PRIOR RESERVOIR CHARACTERIZATION MODEL

The prior pdf for R will be defined through $\int f(r|\theta)f(\theta)d\theta$ to be able to represent the prior uncertainty properly since the expected level of the reservoir characteristics is highly uncertain. A Gaussian model is used here in order to obtain analytic solutions. Extensions to models where only $f(r|\theta)$ comes from the Gaussian family, and $f(\theta)$ is more general, will also be feasible.

Assume that the porosity, $\Phi(x,t)$, can be defined through

$$\begin{aligned} \mathrm{E}\{\Phi(x,t)|\Theta_\Phi = \theta_\Phi\} &= \sum_i f^i_\Phi \cdot \theta^i_\Phi & (3) \\ \mathrm{Cov}\{\Phi(x',t'),\Phi(x'',t'')|\Theta_\Phi = \theta_\Phi\} &= \sigma^2_{\Phi|\Theta}\rho_{\Phi|\Theta}((x',t'),(x'',t'')) & (4) \end{aligned}$$

with Θ_Φ being the Φ-relevant part of Θ, f^i_Φ known spatial trend functions for $\Phi(x,t)$, and $\sigma^2_{\Phi|\Theta}$ and $\rho_{\Phi|\Theta}(\cdot,\cdot)$ the conditional variance and spatial correlation function for $\Phi(x,t)$ respectively.

Similar expressions can be defined for permeability, $K(x,t)$, and reflection coefficients, $C(x,t)$. The dependence between the variables is defined as

$$\mathrm{Cov}\{\Phi(x',t'),K(x'',t'')|\Theta = \theta\} = \sigma_{\Phi K|\Theta}\rho_{\Phi K|\Theta}((x',t'),(x'',t'')) \quad (5)$$

with $\sigma_{\Phi K|\Theta}$ and $\rho_{\Phi K|\Theta}(\cdot,\cdot)$ being the conditional covariance and the spatial cross correlation function. Similar expressions can be defined for the other dependencies.

In compact form this can be written

$$\begin{aligned} \mathrm{E}\{R|\Theta = \theta\} &= F\theta & (6) \\ \mathrm{Cov}\{R|\Theta = \theta\} &= \Sigma_{R|\Theta} & (7) \end{aligned}$$

with $\Theta = (\Theta_\Phi, \Theta_K, \Theta_C)^T$, $F = (F_\Phi, F_K, F_C)^T$ and $\Sigma_{R|\Theta}$ being a covariance matrix defined by the expressions above. By assuming that $(R|\Theta = \theta)$ is a trivariate Gaussian random function, the heterogeneity part of the prior pdf, $f(r|\theta)$, is fully defined. Assume also that Θ is multi-Gaussian with expectation μ_Θ and covariance matrix Σ_Θ, then also $f(\theta)$ is fully defined. It follows that $f(r)$ is a trivariate Gaussian random function with parameters that can be determined from the definitions above. This is a model according to the Bayesian kriging model defined in Omre and Halvorsen (1989).

In order to simplify the notation, the trivariate random function R will be defined as a multivariate random variable in three variables on a grid over the domain $\mathcal{A} \times \mathcal{T}$. This discretized version will also be denoted R.

3.3. OBSERVATION LIKELIHOOD MODEL

The reservoir specific observations consist of well observations and seismic data. In order to simplify the model, linear transfer functions, $g(\cdot,\psi)$, and additive Gaussian error terms are defined. The data acquisition parameter, ψ, can be more generally defined.

The well observations may come from wire line logs, core plugs etc, and can be represented by O_w. The data acquisition model is

$$[O_w|R = r, \Psi_w = \psi_w] = A_w(\psi_w)r + U_w(\psi_w) \tag{8}$$

with ψ_w being the well related acquisition parameters, A_w being a matrix function of ψ_w representing the well data acquisition transfer function and $U_w(\psi_w)$ being a centered multi-Gaussian random variable with covariance matrix $\Sigma_{U_w}(\psi_w)$. The prior pdf of the acquisition parameter, $f(\psi_w)$, should be assigned by petrophysicists.

The seismic observations O_s are considered to be a convolution of the reflection coefficients, $C(\cdot,\cdot)$, and a seismic wavelet containing some unknown parameter ψ_s. Let the convolution be represented by the linear transfer function

$$[O_s|R = r, \Psi_s = \psi_s] = A_s(\psi_s)r + U_s(\psi_s) \tag{9}$$

with ψ_s being the geophysics related acquisition parameters, $A_s(\psi_s)$ being a matrix function of ψ_s representing the convolution and $U_s(\psi_s)$ being a centered multi-Gaussian random variable with covariance matrix $\Sigma_{U_s}(\psi_s)$. The prior pdf of the acquisition parameter, $f(\psi_s)$, should be assigned by geophysicists.

In compact form one may write:

$$[O|R = r, \Psi = \psi] = A(\psi)r + U(\psi) \tag{10}$$

with $O = (O_w, O_s)^T$, $\psi = (\psi_w, \psi_s)^T$, $U(\psi) = (U_w(\psi_w), U_s(\psi_s))^T$, and $A(\psi)$ being a matrix with entries defined by the expressions above. By assuming conditional independence in O_w and O_s, it follows that $U(\psi)$ is a centered multi-Gaussian random variable with covariance matrix $\Sigma_U(\psi)$ also defined by the expressions above. Consequently, the conditional observation likelihood function, $f(o|r,\psi)$, is Gaussian with expectation $A(\psi)r$ and covariance matrix $\Sigma_U(\psi)$. Assuming independence between ψ_w and ψ_s entails that the prior pdf of ψ is $f(\psi) = f(\psi_w)f(\psi_s)$.

3.4. POSTERIOR RESERVOIR CHARACTERIZATION MODEL

The posterior pdf for the reservoir characteristics given the observations is given by (1). Whenever sampling techniques are used, it is more convenient to consider $f(r,\theta,\psi|o)$ with the decomposition:

$$f(r,\theta,\psi|o) = f(r|\theta,\psi,o)f(\theta|\psi,o)f(\psi|o). \tag{11}$$

The first term on the right hand side, $f(r|\theta,\psi,o)$, can be shown to be Gaussian with parameters

$$
\begin{aligned}
&\mathrm{E}\{R|\Theta=\theta,\Psi=\psi,O=o\} = \\
&\qquad F\theta + \Sigma_{R|\Theta}A(\psi)^T\left[A(\psi)\Sigma_{R|\Theta}A(\psi)^T+\Sigma_U\right]^{-1}(o-A(\psi)F\theta) \quad (12)\\
&\mathrm{Cov}\{R|\Theta=\theta,\Psi=\psi,O=o\} = \\
&\qquad \Sigma_{R|\Theta} - \Sigma_{R|\Theta}A(\psi)^T\left[A(\psi)\Sigma_{R|\Theta}A(\psi)^T+\Sigma_U\right]^{-1}A(\psi)\Sigma_{R|\Theta}. \quad (13)
\end{aligned}
$$

The second term, $f(\theta|\psi,o)$, can also be shown to be Gaussian with parameters

$$
\mathrm{E}\{\Theta|\Psi=\psi,O=o\} = \mu_\Theta + \Sigma_\Theta F^T A(\psi)^T\Sigma_{O|\Psi}^{-1}(o-A(\psi)F\mu_\Theta) \quad (14)
$$

$$
\mathrm{Cov}\{\Theta|\Psi=\psi,O=o\} = \Sigma_\Theta - \Sigma_\Theta F^T A(\psi)^T\Sigma_{O|\Psi}^{-1}A(\psi)F\Sigma_\Theta \quad (15)
$$

where

$$
\Sigma_{O|\Psi} = A(\psi)(F\Sigma_\Theta F^T + \Sigma_{R|\Theta})A(\psi)^T + \Sigma_U(\psi). \quad (16)
$$

The third term is the complex one that cannot be fully determined analytically. It can be shown that it takes the form

$$
\begin{aligned}
f(\psi|o) &= \mathrm{const}\times f(o|\psi)f(\psi)\\
&= \mathrm{const}\times|\Sigma_{O|\Psi}|^{-1/2}\exp\left\{-\frac{1}{2}(o-A(\psi)F\mu_\Theta)^T\Sigma_{O|\Psi}^{-1}(o-A(\psi)F\mu_\Theta)\right\}\\
&\qquad\times f(\psi) \quad (17)
\end{aligned}
$$

Sampling from the pdf $f(r,\theta,\psi|o)$ can be done in a sequential manner by firstly sampling a ψ from $f(\psi|o)$, then sampling a θ from $f(\theta|\psi,o)$ and then lastly sampling a r from $f(r|\theta,\psi,o)$. The samples obtained for r will be realizations of the posterior pdf $f(r|o)$, being the objective of the study.

Sampling from $f(\psi|o)$ is not trivial since the pdf can only be analytically determined up to a constant. The sampling has to be based on the Metropolis-Hastings algorithm, see Hegstad et al. (1993), which is an iterative algorithm that converges to the required pdf. The fact that ψ is a low-dimensional vector makes this approach feasible. Note, however, that each iteration involves computation of $\Sigma_{O|\Psi}^{-1}$, i.e. a matrix of dimension equal to the number of observations, and this might require considerable computer resources. Given a realization of Ψ, sampling $f(\theta|\psi,o)$ is trivial since it is Gaussian. Similarly given realizations of Θ and Ψ, sampling from $f(r|\theta,\psi,o)$ is trivial since it entails sampling from a Gaussian random function with given parameters. In the example in the next section this sampling procedure is used.

Whenever samples of $(R, \Theta, \Psi | O = o)$ from $f(r, \theta, \psi | o)$ are available, samples of $(R|O = o)$ from $f(r|o)$ are obtained by considering the R-entries, samples of $(\Theta|O = o)$ from $f(\theta|o)$ by considering the Θ-entries, and samples from $(\Psi|O = o)$ from $f(\psi|o)$ by considering the Ψ-entries.

4. Possible Approximations

In real applications there are two critical factors: The dimensionality of R and the dimensionality of O_s. The former is involved in $f(r|\theta, \psi, o)$ and the latter mostly in $f(\psi|o)$.

Consider $f(r|\theta, \psi, o)$ and let $r = (r^1, \ldots, r^N)$ be r decomposed into an arbitrary sequence of grid nodes over the lateral domain $\mathcal{A}$. Let the seismic part of o be decomposed accordingly, $o_s = (o_s^1, \ldots, o_s^N)$. Consider the factorization

$$
\begin{aligned}
f(r|\theta, \psi, o) &= f(r^N | r^{N-1}, \ldots r^1, \theta, \psi, o) \\
&\times f(r^{N-1} | r^{N-2}, \ldots r^1, \theta, \psi, o) \\
&\vdots \\
&\times f(r^2 | r^1, \theta, \psi, o) \\
&\times f(r^1 | \theta, \psi, o)
\end{aligned}
\tag{18}
$$

which defines the sequential simulation algorithm, see Gómez-Hernández and Journel (1993). For each step in the sequential simulation algorithm one needs to sample from $f(r^i | r^{i-1}, \ldots r^1, \theta, \psi, o_w, o_s^1, \ldots, o_s^N)$, which for really large i can be prohibitive although it is Gaussian, since the conditioning is very high-dimensional. A reasonable approximation would be to only involve a subset of $(r^{i-1}, \ldots r^1)$ and $(o_s^1, \ldots, o_s^N)$ in the conditioning. A natural choice would be to involve only r's and o_s's that are spatially close to the r^i. The corresponding $f(r|\cdot)$ can of course be analytically determined. Approximations along these lines are used in Bortoli et al. (1993) and Haas and Dubrule (1994).

Consider $f(\psi|o)$, which can be written as $f(\psi|o_w, o_s^1, \ldots, o_s^N)$. Here it is again the dimension of the conditioning that is the problem. A reasonable approximation would be to use a representative subset of o_s's as conditioning data. One choice could be to use the o_s's in the location of the o_w's plus a set of o_s in a coarse grid covering $\mathcal{A}$. This can easily be included in the sampling procedure described above.

5. Example

A simple synthetic example is presented in order to demonstrate the approach. The reservoir is a two-dimensional cross section discretized into a

16×16 grid with grid cell size ($50\text{m} \times 2.5\text{ms}$). The reservoir characteristics considered are porosity and reflection coefficients, $R = (\Phi(x,t), C(x,t))$.

The prior pdf of R contains the stochastic model parameters $\Theta = (\Theta_\Phi, \Theta_C)$, with Θ_Φ being a constant expectation of $\Phi(\cdot,\cdot)$ and Θ_C being a constant expectation of $C(\cdot,\cdot)$. The conditional variances are $\sigma^2_{\Phi|\Theta} = 0.05^2$ and $\sigma^2_{C|\Theta} = 0.025^2$ with covariance $\sigma_{\Phi C|\Theta} = -6.25 \cdot 10^{-4}$. All spatial correlation functions are of the form $\rho(\Delta) = \exp\{-2.5 \cdot 10^{-3} \cdot \Delta\}$ with Δ being a distance measure in the (x,t)-reference domain with anisotropy factor $1 : 8 \cdot 10^5$. This defines $f(r|\theta)$. The prior pdf of the model parameters, $f(\theta)$, is bi-Gaussian with variance 0.05^2 for both variables and correlation -0.5.

Based on this prior pdf, $f(r)$, three realizations are generated, see Figure 1. These realizations reflect the uncertainty in the prior information.

The likelihood function is defined for two types of well information and seismic data. There are two wells in the cross section. In each well the porosity and reflection coefficients $(\Phi(\cdot,\cdot), C(\cdot,\cdot))$, are observed without averaging and with independent Gaussian error such that Σ_{U_w} is a diagonal matrix with variances 0.02^2. There are no unknown data acquisition parameters in the wells, hence $f(o_w|r)$ is fully defined. The seismic data for each lateral coordinate consist of the vertical reflection coefficients convolved with a Ricker wavelet having data acquisition parameter $\Psi = (\nu_M)$. Hence the entries in $A_s(\psi)$ are based on the wavelet $h(t) = (1 - 2\pi^2\nu_M^2 t^2)e^{-\pi^2\nu_M^2 t^2}$ where the peak frequency ν_M determines the pulse width. The prior pdf of the Ricker wavelet parameter $f(\psi_s)$ is Gaussian with expectation 50 and variance 10^2. The observation errors are independent Gaussian such that Σ_{U_s} is a diagonal matrix with variances 0.02^2. This defines $f(o_s|r,\psi_s)$ and the prior pdf $f(\psi_s)$. The observation likelihood $f(o|r) = \int f(o|r,\psi) f(\psi) d\psi$ is then fully defined.

The latter of the three realizations from the prior is considered to be the 'true reservoir'. The observed values o are taken from this realization according to the description above. The well observations, o_w, and the seismic data, o_s, are displayed in Figure 1 below the 'true reservoir'. The expected values of the porosity and the reflection coefficients in this particular realization are $\theta^{\text{True}} = (\theta_\Phi^{\text{True}}, \theta_C^{\text{True}}) = (0.24, 0.07)$. The wavelet parameter ν_M is $\psi_s^{\text{True}} = \nu_M^{\text{True}} = 50.8$.

From the analytical expressions of the posterior pdf, $f(r,\theta,\psi|o)$, and the sampling procedure described in the previous section, samples of the posterior $(R, \Theta, \Psi | O = o)$ can be generated. By considering the the Ψ-entry, the posterior for the wavelet parameter $\psi = \nu_M$, i.e. $f(\psi|o)$, is sampled. The result is presented in the bottom left in Figure 1. By considering the Θ-entry, the posterior for the expected values of the porosity and the reflection coefficients $\Theta = (\Theta_\Phi, \Theta_C)$, i.e. $f(\theta|o)$, is sampled. The result is presented in

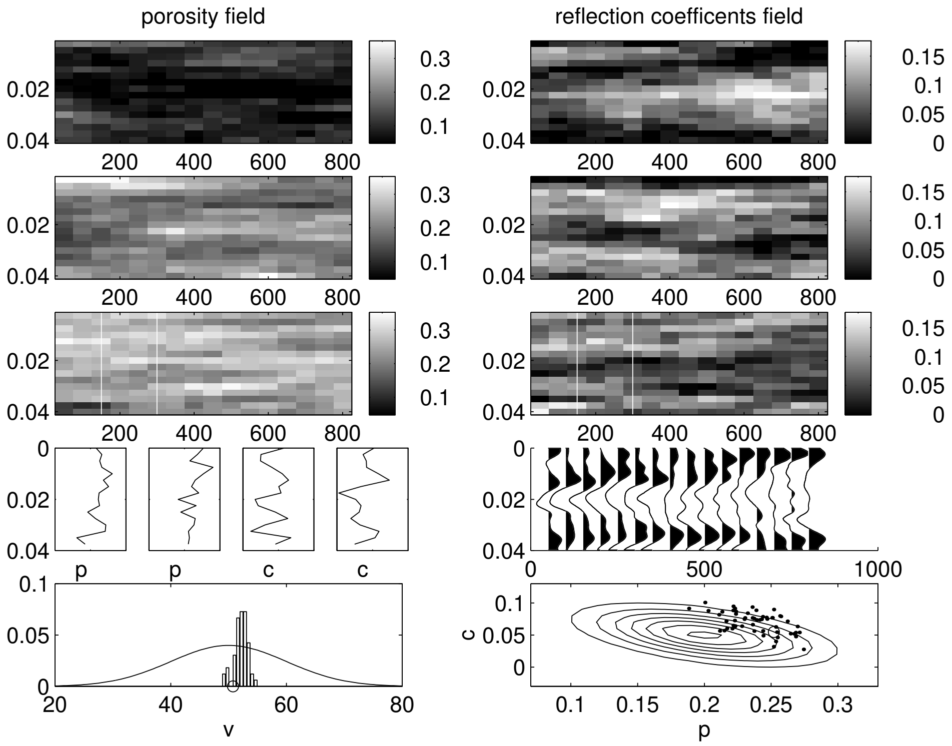

Figure 1. Simulation example for Gaussian random field. From above: The three first rows show porosity and reflectivity fields sampled from the prior distribution $f(r)$. Row no. 3 is taken to be the 'true' reservoir. Vertical lines indicate position of wells. Row no. 4 shows observations in wells and the seismic observations from the above reservoir. Row no. 5 displays prior and posterior distribution of data acquisition parameter Ψ and model parameters Θ. The true values are marked with an 'o'.

the bottom right in Figure 1. In these figures the prior pdf's $f(\psi)$ and $f(\theta)$ are also displayed, and so are the parameter values of the 'true reservoir'. Note that the center of the posterior pdf is shifted towards the true values and that the precision is increased due to the conditioning on the reservoir specific observations.

In Figure 2, three realizations of the posterior reservoir characteristics given the reservoir specific observations are presented. They represent the uncertainty in $f(r|o)$ and are obtained by considering the R-entries in the samples. Note that the well observations are reproduced up to the precision given and that the seismic data are constraining the realizations. The dispersion between realizations will be less than the dispersion between the realizations that are presented in Figure 1. Note in particular how the

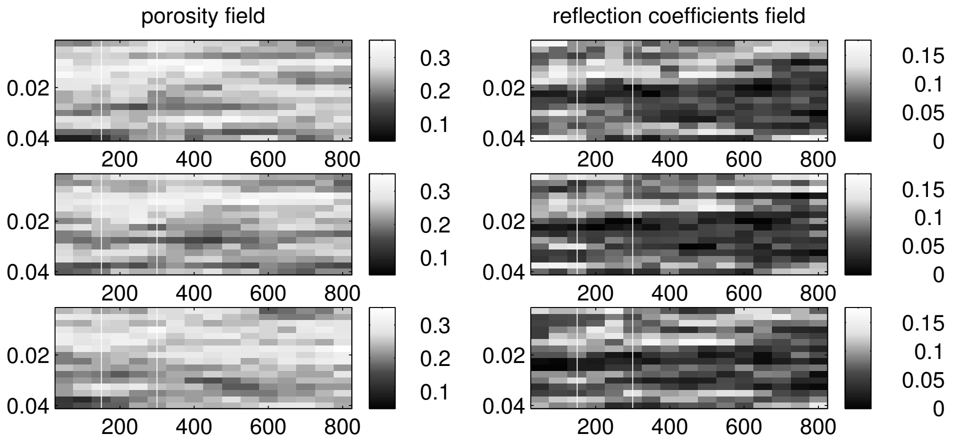

Figure 2. Simulation example for Gaussian random field conditioned on seismic and well data. The figure shows three realizations from the posterior distribution of $R|O=o$.

posterior of the model parameters, Θ, tends to stabilize the level of the reservoir characteristics in the posterior.

In Figure 3, the seismic responses to prior and posterior models are presented. The left column contains seismic responses from three realizations from the prior model. The third one is taken to be the true one, and the observed seismic data is displayed below. The difference is due to measurement error. The correlations between the observed seismic data and seismic responses from the priors vary considerably. The right column contains seismic responses from realizations from the posterior model given well observations and the observed seismic data. The correlations are now close to one, which is as expected. The true Ricker wavelet is displayed below.

6. Closing remarks

A spatial model for the reservoir characteristics and explicit models for the data acquisition procedures are merged in a Bayesian framework to define a model for the reservoir characteristics. The major contributions are:

- A formally correct posterior model for the reservoir characteristics, with a strong Gaussian flavor, given well and seismic observations has been developed. This allows correct assessments of uncertainties.
- Simultaneous simulations of reservoir characteristics and estimation of parameters in the data acquisition model has been made possible.

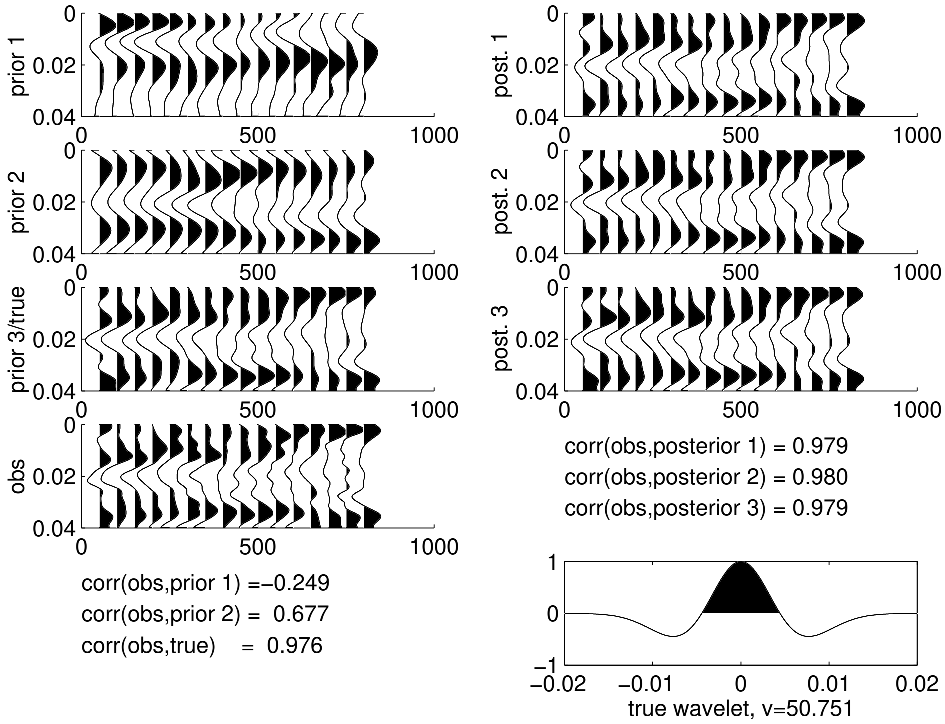

Figure 3. Comparison of seismic observations from prior and posterior models.

- A framework for the integration of seismic data in stochastic reservoir characterization has been developed. It can easily can extended to more complex models.
- The approach is simple to implement and obvious approximations can be seen. Hence the approach is easily applicable to real problems.

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