BAYESIAN DISJUNCTIVE KRIGING APPLIED TO PREDICTION OF RESERVOIR VOLUME

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Abstract. The theory of disjunctive kriging is extended to cover models of the Bayesian kriging type. The Bayesian disjunctive kriging predictor is applied to evaluate the volume of a petroleum reservoir, and compared to a predictor based on full Gaussian assumptions.

1. Introduction

Geostatistical methodology is extensively used in evaluation of petroleum reservoirs, see Haldorsen and Damsleth (1990). Precise mapping of the outline of the reservoir is of utmost importance. Fortunately, subsurfaces like the cap rock and other distinct horizons are often seismic reflectors. Hence the outline often can be predicted based on seismic reflection times and depth observations in a low number of wells. The depth conversion of the seismic reflection times remain as a major source of uncertainty. In Abrahamsen et al. (1991) and Abrahamsen and Omre (1994) geostatistical techniques in a Bayesian setting are used for seismic depth conversion.

Let the depth to a subsurface be represented by a random function $\{Z(x);\, x \in \mathcal{D}\}$ with x being a reference variable in $\mathcal{D} \subset \mathbf{R}^2$. There are several objectives of mapping a subsurface, some are:

- prediction of depth at a new location for drilling, i.e. $Z(x_0)$.
- prediction of the rock volume below the subsurface and above a hydrocarbon contact z_c, i.e. $V_r = \int_{\mathcal{D}}(z_c - Z(u))I\{Z(u) \leq z_c\}du$ with $I\{A\}$ being an indicator with value 1 if A is true and zero otherwise.

- prediction of production of hydrocarbons; i.e. $Q(t) = v(\{Z(x); x \in \mathcal{D}\}, \ldots)$ with $v(\cdot)$ being a fluid flow simulator where the subsurface is just one out of many input variables.

These predictions have to be based on a stochastic model for $\{Z(x);\ x \in \mathcal{D}\}$. This article focuses on the prediction of the volumetrics, V_r, in a Bayesian setting under minimum assumptions. This leads to the development of a Bayesian disjunctive kriging predictor being a generalization of the disjunctive kriging predictor, see Matheron (1976) and Rivoirard (1994). Under the same conditions an indicator based predictor can be defined, see Kolbjørnsen (1996). This will be in the spirit of indicator kriging, see Journel (1983), but it is not presented here.

2. Model assumptions

Consider the continuous random function, $\{Z(x);\ x \in \mathcal{D}\}$, representing depth to a subsurface. The function is defined through a set of stochastic parameters $\underline{B} = (B_1, \ldots, B_M)^T$ being related to seismic velocity and a set of known functions $\underline{f}(x) = (f_1(x), \ldots, f_M(x))^T$ being related to seismic reflection times. The conditional moments are,

$$\mu_{Z|B}(x) = E\{Z(x)|\underline{B} = \underline{b}\} = \underline{f}^T(x) \cdot \underline{b} \qquad ; x \in \mathcal{D} \tag{1}$$

$$\sigma_{ZZ|B}(x', x'') = \mathrm{Cov}\{Z(x'), Z(x'')|\underline{B} = \underline{b}\} = \sigma^2_{Z|B} \cdot \rho_{Z|B}(x' - x'') \quad ; x', x'' \in \mathcal{D}$$

with $\sigma^2_{Z|B}$ and $\rho_{Z|B}(\cdot)$ being the variance and spatial correlation function in a stationary residual random function. Hence the seismic data are considered to be related to the expected depth to the subsurface. The moments of the stochastic model parameters are defined by $E\{\underline{B}\} = \underline{\mu}_B$ and $\mathrm{Var}\{\underline{B}\} = \sum_B$, they represent a prior model for seismic velocity. This entails that the marginal moments are:

$$\begin{aligned}
\mu_Z(x) = E\{Z(x)\} &= \underline{f}^T(x) \cdot \underline{\mu}_B \quad ; x \in \mathcal{D} \\
\sigma_{ZZ}(x', x'') &= \mathrm{Cov}\{Z(x'), Z(x'')\} \\
&= \sigma^2_{Z|B}\rho_{Z|B}(x' - x'') + \underline{f}^T(x') {\textstyle\sum_B} \underline{f}(x'') \qquad ; x', x'' \in \mathcal{D} \\
\sigma^2_Z(x) = \mathrm{Var}\{Z(x)\} &= \mathrm{Cov}\{Z(x), Z(x)\} \quad ; x \in \mathcal{D}.
\end{aligned} \tag{2}$$

These assumptions correspond to the basic assumptions of Bayesian kriging being simultaneously and independently developed in Kitanidis (1986) and Omre (1987), and later developed into a formalism for seismic depth conversion in Omre and Halvorsen (1989). Recall that the two extremes of Bayesian kriging represent simple and universal kriging respectively, hence it bridges the gap between these kriging models.

Let the random function be observed in N locations $x_1, \ldots, x_N \in \mathcal{D}$, defining :

$$\underline{Z}^d = (Z(x_1), \ldots, Z(x_N)).$$

Consider prediction of $Z(x_0)$ from $\underline{Z}^d = \underline{z}^d$. By enforcing a linear predictor, $\sum_i^N \alpha_i Z(x_i)$ with unknown weights $\underline{\alpha} = (\alpha_1, \ldots, \alpha_N)$ to be determined, and the minimizing criterion being a linear combination of terms of form $E\{Z(x')Z(x'')\}$, then the assumptions defined above are sufficient. The solution corresponds to the Bayesian kriging predictor.

Consider prediction of $V_r = \int_{\mathcal{D}} (Z(u) - z_c) I\{Z(u) \geq z_c\} du = \int_{\mathcal{D}} g_t(Z(u)) du$ from $\underline{Z}^d = \underline{z}^d$. By enforcing a predictor of the form $\sum_i^N \nu_i(Z(x_i))$ with unknown functions $\underline{\nu} = (\nu_1(\cdot), \ldots, \nu_N(\cdot))$ to be determined, and minimizing criterion being a linear combination of terms of form $E\{g(Z(x'), Z(x''))\}$ with $g(\cdot, \cdot)$ being arbitrary functions, then additional model assumptions must be made. More specifically, the bivariate characteristics $(Z(x'), Z(x''))$; all $x', x'' \in \mathcal{D}$ must be known. In disjunctive kriging, predictors of this type have been defined whenever the model is of simple kriging type. This is a model type of very limited interest when predicting volumetrics. Since the effect of the residual random function is averaged out, the integral is then only a function of known model parameters of the expectation. There exist no disjunctive kriging solution for models of the general universal kriging type. In the following section a predictor for models of the Bayesian kriging type will be developed, hence the model may contain partly known model parameters.

Consider prediction of $Q(t) = v(\{Z(x); x \in \mathcal{D}\}, \ldots)$ from $\underline{Z}^d = \underline{z}^d$. This requires the knowledge of the full probability structure of the random function, and hence a predictor of any complexity and any criterion can be defined. This assumption corresponds to the assumption required whenever a Monte Carlo simulation approach is applied.

3. Bayesian Gaussian-related predictors

Consider prediction of the volumetrics which can be written as $V_r = \int_{\mathcal{D}} g_t(Z(u)) du$ with $g_t(\cdot)$ being a known target function. In the developments the target function will be denoted $h(Z(x_0)) = h(Z^0)$ for an arbitrary x_0. The results will be valid for linear combinations, or integrals, of target functions as well.

Firstly, the Bayesian disjunctive kriging predictor based on assumptions about the bivariate properties only, will be developed. Secondly, the Bayesian Gaussian predictor based on assumptions about the full probability structure of the random function will be presented.

3.1. BAYESIAN DISJUNCTIVE KRIGING PREDICTOR

The predictor is defined to be of the form:

$$\hat{h}^{\mathcal{D}}(Z^0) = \sum_{i=1}^{N} \nu_i(Z(x_i)) = \sum_i \nu_i(Z^i) \tag{3}$$

with $\underline{\nu} = (\nu_i(\cdot), \ldots, \nu_N(\cdot))$ being unknown functions to be determined. The minimizing criterion is:

$$\hat{\underline{\nu}} = \arg\min_{\underline{\nu}} E\{[h(Z^0) - \sum_i \nu_i(Z^i)]^2\}. \tag{4}$$

Hence $\nu_i(\cdot), i = 1, \ldots, N$ has to be such that the criterion is defined. According to standard projection theory, see Billingsley (1995), the best predictor under expected squared error loss is the one having prediction error orthogonal to the space spanned by the predictor. This provides,

$$E\{(h(Z^0) - \sum_{i=1}^{N} \nu_i(Z^i)) \cdot \sum_{j=1}^{N} \kappa_j(Z^j)\} = 0 \qquad ; \text{all } \kappa_j(\cdot); j = 1, \ldots, N$$

$$\Updownarrow$$

$$E\{h(Z^0) \cdot \kappa_j(Z^j)\} = \sum_{i=1}^{N} E\{\nu_i(Z^i)\kappa_j(Z^j)\} \qquad ; j = 1, \ldots, N; \text{all } \kappa_j(\cdot) \tag{5}$$

with $\kappa_j(\cdot)$ being an arbitrary function of the same type as $\nu_j(\cdot)$ for $j = 1, \ldots, N$.

In order to solve this set of equations the bivariate probability density function, pdf, of $(Z(x'), Z(x''))$; all $x', x'' \in \mathcal{D}$ has to be known. Assume that,

$$(Z(x'), Z(x'')) \rightarrow \text{bi-Gauss}\left[\begin{pmatrix} \mu_Z(x') \\ \mu_Z(x'') \end{pmatrix}, \textstyle\sum_{Z'Z''}\right]; \quad \text{all } x', x'' \in \mathcal{D} \tag{6}$$

with $\sum_{Z'Z''}$ having elements $\sigma_{ZZ}(x', x'')$ defined by expression (2). Hence the univariate pdf, $f_Z(z)$, is Gauss$(\mu_Z(x), \sigma_Z^2(x))$. A consequence of this assumption is that $\underline{B}$ is multi-Gauss$(\underline{\mu}_B, \sum_B)$.

The isofactorial model for the general bi-Gaussian case, can be shown to be:

$$f_{Z'Z''}(z', z'') = \sum_{k=0}^{\infty} \rho_{Z'Z''}^k \eta_k'(z') \eta_k''(z'') f_{Z'}(z') f_{Z''}(z'') \text{ ; all } z', z'' \tag{7}$$

with

$$\rho_{Z'Z''} = \frac{\sigma_{ZZ}(x', x'')}{\sigma_Z(x') \cdot \sigma_Z(x'')} \qquad ; x', x'' \in \mathcal{D}$$

$$\eta_k'(z) = \frac{1}{\sqrt{k!}} H_k\left(\frac{z - \mu_Z(x')}{\sigma_Z(x')}\right) \qquad ; k = 0, 1, 2, \ldots$$

and the Hermitian polynomials of order k being, see Journel and Huijbregts (1978):

$$H_k(u) = e^{u^2/2}\frac{d^k}{du^k}\left(e^{-u^2/2}\right).$$

The polynomials $\eta_k(\cdot); k = 0, 1, 2, \ldots$ constitute an orthonormal basis with respect to $f_Z(\cdot)$. Note that this is a generalization of the isofactorial model used in traditional disjunctive kriging since it allows location dependent expectations and variances, see Rivoirard (1994).

Since the Hermitian functions are polynomials, they span the space of all functions for which the criterion (4) is defined. Hence the known target function can be expressed as,

$$h(z) = \sum_{l=0}^{\infty} a_l \eta_l^0(z)$$

with

$$\eta_l^0(z) = \frac{1}{\sqrt{l!}} \cdot H_l\left(\frac{z - \mu_Z(x^0)}{\sigma_Z(x^0)}\right) \qquad ; l = 0, 1, 2, \ldots$$

and the known coefficients $\underline{a} = (a_0, \ldots)$ being defined by:

$$a_l = \int_{-\infty}^{\infty} h(u)\eta_l^0(u) f_{Z^0}(u)du \qquad ; l = 0, 1, 2, \ldots$$

Similarly, the unknown terms in the predictor can be expressed as,

$$\nu_i(z) = \sum_{l=0}^{\infty} \alpha_{il}\eta_l^i(z) \qquad ; i = 1, \ \ldots, N$$

with $\eta_l^i(\cdot)$ being defined according to $\eta_l^0(\cdot)$. The set of unknown coefficients $\underline{\alpha_l} = (\alpha_{1l}, \ldots, \alpha_{Nl})$; $l = 0,\ 1,\ 2, \ldots$ are to be determined. Correspondingly the functions $\kappa_j(\cdot); j = 1, \ldots, N$ defined in expression (5) can be expressed,

$$\kappa_j(z) = \sum_{l=0}^{\infty} \beta_{jl}\eta_l^j(z) \qquad ; j = 1, \ldots, N$$

and for all possible $\kappa_j(\cdot)$ would entail for any choices of weights $\underline{\beta_l} = (\beta_{1l}, \ldots, \beta_{Nl})$; $l = 0,\ 1,\ 2, \ldots$Expression (5) can then be written as:

$$\begin{aligned}
E\{\sum_{l=0}^{\infty} a_l\eta_l^0(Z^0)\sum_{k=0}^{\infty}\beta_{jk}\eta_k^j(Z^j)\} &= \sum_{i=1}^{N} E\{\sum_{l=0}^{\infty}\alpha_{il}\eta_l^i(Z^i)\sum_{k=0}^{\infty}\beta_{jk}\eta_k^j(Z^j)\} \\
&\quad ; j = 1, \ldots, N \quad ; \text{all}\underline{\beta_k}; k = 0,\ 1, \ldots \\
&\Downarrow \\
\sum_{l=0}^{\infty} a_l E\{\eta_l^0(Z^0)\eta_k^j(Z^j)\} &= \sum_{i=1}^{N}\sum_{l=0}^{\infty}\alpha_{il}E\{\eta_l^i(Z^i)\eta_k^j(Z^j)\} \\
&\quad ; j = 1, \ldots, N \qquad ; k = 0,\ 1, \ldots
\end{aligned}$$

$$\Downarrow$$

$$a_k \rho^k_{Z^0 Z^j} = \sum_{i=1}^{N} \alpha_{ik} \rho^k_{Z^i Z^j} \qquad ; j = 1, \ldots, N \qquad ; k = 0, 1, \ldots \tag{8}$$

The first implication follows from the requirement that all coefficients of $\underline{\beta}_k$ must be identical in order to ensure equality for all $\underline{\beta}_k$. The second implications follows from the fact that $E\{\eta^i_l(Z^i)\eta^j_k(Z^j)\} = \rho^k_{Z^i Z^j} \cdot \delta_{lk}$. This can be seen from using the definition of expectation with respect to expression (7) and the orthonormality properties of $\eta^i_l(\cdot)$.

From the expression above the vectors of weights $\underline{\hat{\alpha}}_k; \quad k = 1, 2, \ldots$ can be determined for each k separately. Note that for $k = 0$, $a_0 = \sum_i \alpha_{i0}$, and hence it is not possible to determine $\underline{\alpha}_0$. This creates no problems since $\eta^i_0(z) = 1$ all i, which entails that only the sum appears in the relevant expressions.

Hence the Bayesian disjunctive kriging predictor is defined as:

$$\hat{h}^D(Z^0) = \sum_{i=1}^{N} \hat{\nu}_i(Z^i) = \sum_{i=1}^{N} \sum_{l=0}^{\infty} \hat{\alpha}_{il} \eta^i_l(Z^i) \tag{9}$$

It can easily be shown that this predictor is unbiased, hence the corresponding prediction variance, being identical to criterion (4), is:

$$\begin{aligned} \mathrm{Var}\{h(Z^0) - \hat{h}^D(Z^0)\} &= E\{(h(Z^0) - \hat{h}^D(Z^0))^2\} \\ &= \sum_{l=0}^{\infty} a_l [a_l - \sum_{i=1}^{N} \hat{\alpha}_{il} \rho^l_{Z^0 Z^i}] \end{aligned} \tag{10}$$

3.2. BAYESIAN GAUSSIAN PREDICTOR

The predictor is defined to be of general form,

$$\hat{h}^G(Z^0) = \nu(Z(x_1), \ldots, Z(x_N)) = \nu(\underline{Z}^d) \tag{11}$$

with $\nu(\cdot)$ being an unknown function to be determined. The criterion is also here chosen to be the expected squared prediction error, hence the best predictor is known to be the conditional expectation:

$$\hat{h}^G(Z^0) = E\{h(Z^0)|\underline{Z}^d\} = \hat{\nu}(\underline{Z}^d). \tag{12}$$

In order to solve this, the full probability structure of the random function $\{Z(x);\, x \in \mathcal{D}\}$ has to be known. Assume that it is a Gaussian random function which entails;

$$(Z(x_1), \ldots, Z(x_k)) \rightarrow f_{Z^1 \ldots Z^k}(z^1, \ldots, z^k)\text{- multi-Gaussian} \tag{13}$$

for all k and all configurations $(x_1, \ldots, x_k)$. The appropriate parameters in the multi-Gaussian pdf can be determined from expression (2).

Hence the Bayesian Gaussian prediction is defined as:

$$\hat{h}^G(z^0) = \int_{-\infty}^{\infty} h(u) f_{Z^0|Z^1 \cdots Z^N}(u|z^1, \ldots, z^N) du = \hat{\nu}(\underline{z}^d). \tag{14}$$

The corresponding conditional prediction variance is:

$$\begin{aligned} &\mathrm{Var}\{h(Z^0) - \hat{h}^G(Z^0)|\underline{Z}^d = \underline{z}^d\} = \\ &\qquad \int_{-\infty}^{\infty} (h(u) - \hat{h}^G(z^0))^2 f_{Z^0|Z^1 \cdots Z^N}(u|z^1, \ldots, z^N) du. \end{aligned} \tag{15}$$

These expressions must for most functions $h(\cdot)$ be determined numerically.

Note that in this case the conditional prediction variance for the set of observations actually made can be determined. The prediction variance for the Bayesian disjunctive kriging case, see expression (10) on the other hand, can only be determined as randomized over all possible outcomes of the observations. Since $\hat{h}^G(\cdot)$ is obtained by projection into a larger space of predictors than $\hat{h}^D(\cdot)$, the following order relation will hold:

$$\mathrm{Var}\{h(\cdot) - \hat{h}^G(\cdot)\} \leq \mathrm{Var}\{h(\cdot) - \hat{h}^D(\cdot)\} \tag{16}$$

This may not hold whenever the prediction variance assosiated with $\hat{h}^G(\cdot)$ is conditioned on the actual observations.

4. Prediction of reservoir volumetrics

The example is identical to the one presented in Abrahamsen et al. (1991). It is based on a cross-section of a Norwegian offshore petroleum reservoir, but the extension to a full three dimensional case is trivial.

The case contains two subsurfaces $\{Z_t(x);\ x \in \mathcal{D}\}$ and $\{Z_b(x);\ x \in \mathcal{D}\}$, corresponding to depth to top and base of the reservoir. The objective is to predict the reservoir volume between these surfaces and above the hydrocarbon/water contact level, z_c, i.e.,

$$V(z_0) = \int_{\mathcal{D}} \left[(z_c - Z_t(u)) I\{Z_t(u) \leq z_c\} - (z_c - Z_b(u)) I\{Z_b(u) \leq z_c\}\right] du \tag{17}$$

see the hatched area in figure 1. This is of the form of expression (3), and can be predicted along the lines described in the previous section.

The stochastic model is according to Abrahamsen et al. (1991):

$$E\{Z_t(x)|\underline{B} = \underline{b}\} = \left[b_t^1 + b_t^2(t_t(x) - t_t^0)\right] t_t(x) \quad ; x \in \mathcal{D}$$

$$\begin{aligned}
\mathrm{Cov}\{Z_t(x'), Z_t(x'')|\underline{B} = \underline{b}\} &= \sigma^2_{t|B} \cdot \rho(x' - x'') \quad ; x',\, x'' \in \mathcal{D} \\
E\{Z_b(x)|\underline{B} = b\} &= [b^1_t + b^2_t(t_t(x) - t^0_t)]\, t_t(x) + b_r\,[t_b(x) - t_t(x)] \quad ; x \in \mathcal{D} \\
\mathrm{Cov}\{Z_b(x'), Z_b(x'')|\underline{B} = \underline{b}\} &= \sigma^2_{b|B} \cdot \rho(x' - x'') \quad ; x',\, x'' \in \mathcal{D} \\
\mathrm{Cov}\{Z_t(x'), Z_b(x'')|\underline{B} = \underline{b}\} &= \sigma_{tb|B}\rho(x' - x'') \quad ; x',\, x'' \in \mathcal{D}
\end{aligned}$$

with $\underline{B} = (B^1_t, B^2_t, B_r)$ being parameters associated with seismic velocities; $\{(t_t(x), t_b(x));\ x \in \mathcal{D}\}$ being observed one-way seismic travel times to top and base reservoir respectively; t^0_t being $\min_x\{t_t(x)\}$; $\sigma^2_{t|B} = 500$ and $\sigma^2_{b|B} = 600$ being the conditional variances for top and base; $\sigma_{tb|B} = 400$ being conditional covariance between top and base; and $\rho(\Delta) = \exp\{-3\Delta/1000\}$ being the spatial correlation function.

The stochastic model parameters, $\underline{B}$, have a prior with uncorrelated elements, expectation $\underline{\mu}_B = (2400, 1500, 2000)$ and variance $\sigma^2_{\underline{B}} = (50^2,\ 750^2,\ 400^2)$. In figure 1, the prior expectations of depth to top and base of the reservoir are presented together with the available depth observations in wells, which also are listed in table 1. There are two vertical and two deviated wells. No velocity observations are available. A contact level $z_c = 4000$ is also marked. This model is of the kind defined in the previous section. By assuming bi-Gaussian relations, the Bayesian disjunctive kriging predictor for the volumetrics, $\hat{V}^D(z_c)$, can be defined. If assumptions of full Gaussian random functions are made, the Bayesian Gaussian predictor, $\hat{V}^G(z_c)$, can be defined.

The computation of $\hat{V}^D(\cdot)$ is done by discretizing the cross section into 111 intervals and by using 20 terms in the Hermite expansion. This is found not to significantly influence the solution. The computation of $\hat{V}^G(\cdot)$ is done along the same lines, but under stricter condictions, see Kolbjørnsen (1996). The processing requirements for the two predictors are almost identical.

The two different predictions for the volumetrics $V(z_c)$ are compared for varying values for the hydrocarbon/water contact levels, z_c, and for different number of wells.

In figure 2 one observes that the presence of one well dramatically improves the prediction of $V(\cdot)$, since the model parameters, $\underline{B}$, are better identified. For one well the predictors are almost identical, because in this case both predictors draw primarily on the bivariate characteristics. When more wells are available, $\hat{V}^D(\cdot)$, is only utilizing bi-Gaussian properties while $\hat{V}^G(\cdot)$ is utilizing the full multi-Gaussian relations. For these cases the higher order interactions contribute with more and hence the predictions do deviate more.

In figure 3 the prediction standard deviations are presented. Note that the figures are not directly comparable since one is unconditioned and the other is conditioned on the observed values. Hence the order relations need not hold. With a shallow contact level z_c, both prediction and prediction error are close to zero since the probability for the top reservoir to intersect the contact level is small. On the other hand, with a deep contact level z_c, the prediction error is primarily linked to the uncertainty in the model parameter B_r which represents the seismic velocity in the reservoir.

In figure 4 the predicted reservoir thickness above $z_c = 4000$ is displayed. The prediction standard deviations are differently defined, as in figure 3. The order relations are not obeyed. One observes that in locations of vertical wells, the predictions coincide and the prediction error is zero. This is not the case for deviation wells since then the depth to either the top or the base is uncertain. Also here one observes that the influence of the basic assumptions for the predictors increases with increasing number of wells.

Figure 5 contains the prediction standard deviations $\mathrm{Sd}\{V(z_c) - \hat{V}^D(z_c)\}$ and $\mathrm{Sd}\{V(z_c) - \hat{V}^G(z_c)\}$ which are comparable, for the cases in figure 4. The latter is randomized over all outcomes of the well observations. In the case with one well only the bivariate characteristics are used and the prediction error is equal up to numerical precision. With four wells, however, the difference is considerable and the order relation holds.

5. Conclusions

The study has demonstrated that:

- a disjunctive kriging predictor for models corresponding to Bayesian kriging models can be defined. This entails that disjunctive kriging predictors exist for models which are intermediate between simple and universal kriging models, including the former and excluding the latter.
- the volume in a reservoir above a contact level can be predicted by non-linear predictors of Bayesian disjunctive kriging type. The predictions based on this model do not vary considerably from predictions based on full Gaussian random functions.
- the prediction variance of the Bayesian disjunctive kriging predictor is not conditioned on the observation values, hence only dependent on the configuration of observation locations. With full Gaussian random function assumptions the conditional prediction variances can be determined.
- Comparing the prediction variances for Bayesian disjunctive kriging predictors and Gaussian random function predictors shows that the latter is considerably smaller. Hence assuming a full Gaussian random function model without the possibility to verify it beyond bivariate

level may cause severe underestimation of the prediction error.

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References

Abrahamsen, P. and Omre, H. (1994) Random Functions and Geological Subsurfaces, *Departments of Mathematics,The University of Trondheim,Norway*, **Statistics no. 3**.

Abrahamsen, P. , Omre, H. and Lia, O. (1991) Stochastic Models for Seismic Depth Conversion of Geological Horizons, *Presented at the Offshore Europe Conference in Aberdeen, 3-6 Sept.* **SPE 23138**.

Billingsley, P. (1995) *Probability and Measure, Third edition*. John Wiley & Sons, New York.

Haldorsen, H.H. , Damsleth, E. (1990) Stochastic Modelling. *JPT* **April** *pp. 404–412.*

Journel, A.G. and Huijbregts, Ch.J. (1978) *Mining Geostatistics* Academic Press, London.

Journel, A.G. (1983) Nonparametric estimation of spatial distributions *Mathematical Geology* **Vol.15** *pp. 445–468.*

Kitanidis, P.K. (1986) Parameter uncertainty in estimation of spatial functions: Bayesian analysis. *Water Resources Research*, **Vol.22, no. 4** *pp. 499–507.*

Kolbjørnsen, O. (1996) Nonlinear Predictors in Random Fields. *Master of science thesis, Department of Mathematical Sciences, Norwegian University of Science and Technology, in Norwegian.*

Matheron, G. (1976) A simple substitute for conditional expectation: The disjunctive kriging. *Advanced Geostatistics in the Mining Industry, ed: M. David, and C. Huijbregts,* Reidel, Dordrecht *pp. 221–236.*

Omre, H. (1987) Bayesian Kriging-Merging Observations and Qualified Guesses in Kriging. *Mathematical Geology,***Vol. 19, no. 1** *pp. 25–39.*

Omre, H. and Halvorsen, K.B. (1989) The Bayesian Bridge Between Simple and Universal Kriging, *Mathmatical Geology,***Vol.21, no. 7** *pp. 767–786.*

Rivoirard, J. (1994) *Introduction to Disjunctive Kriging and non-linear geostatistics*, Clarendon Press,Oxford.

TABLE 1. Well observations

Well	Subsurface	Location [km]	Depth [m]
1	top	400.8	3835
	base	400.8	3956
2	top	402.3	3860
	base	402.3	4006
3	top	403.2	3945
	base	404.0	4145
4	top	399.6	4008
	base	399.2	4174

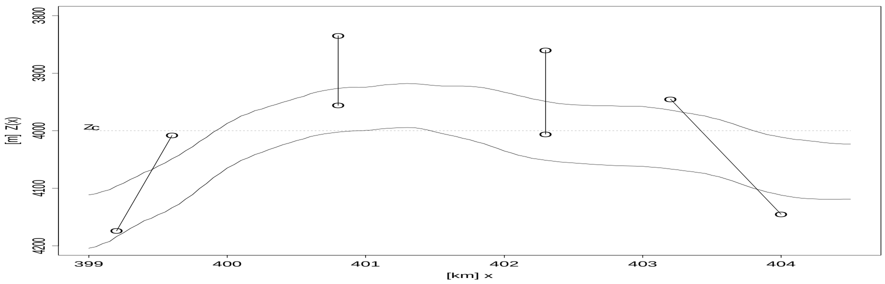


Figure 1. Prior prediction of depths, well observations and contact level z_c.

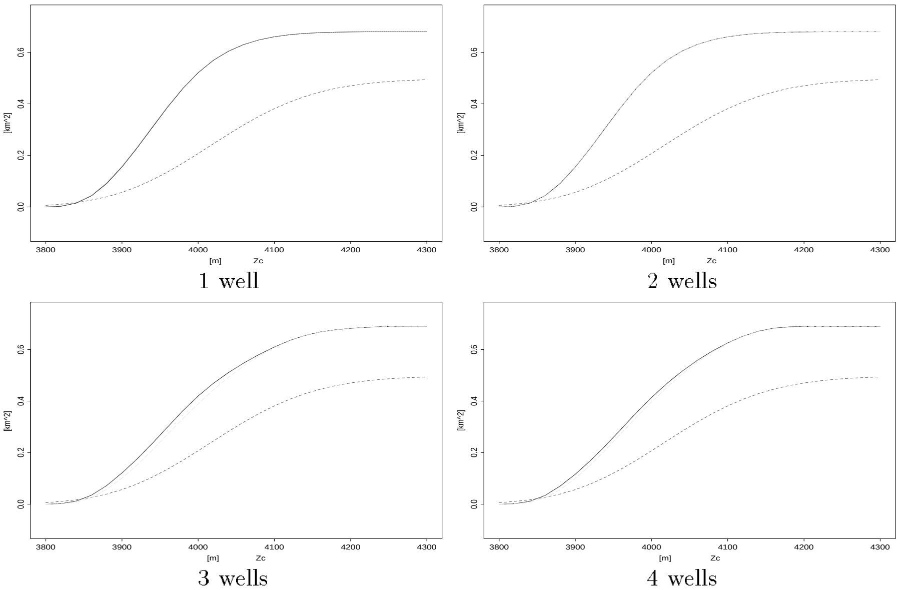
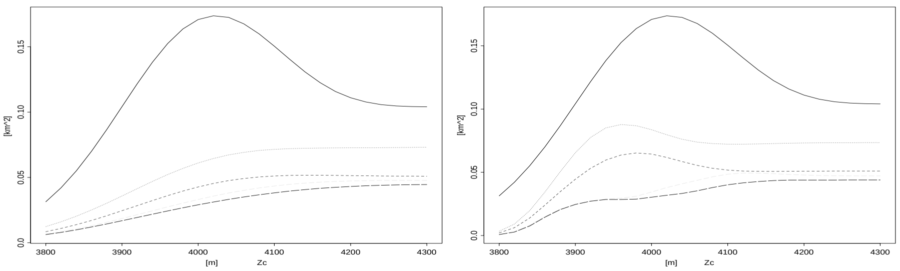


Figure 2. Prediction of $V(z_c)$ as function of z_c. $\cdots$ prior model, i.e. 0 wells; —— $\hat{V}^D(\cdot)$; - - -$\hat{V}^G(\cdot)$.



Bayesian disjunctive kriging predictior $\mathrm{sd}\{V(\cdot) - \hat{V}^D(\cdot)\}$

Bayesian Gaussian predictor $\mathrm{sd}\{V(\cdot) - \hat{V}^G(\cdot)|\underline{Z}^d = \underline{z}^d\}$

Figure 3. Prediction standard deviations as function of z_c; Prior model,i.e. 0 wells; 1 well;2 wells; 3 wells; 4 wells

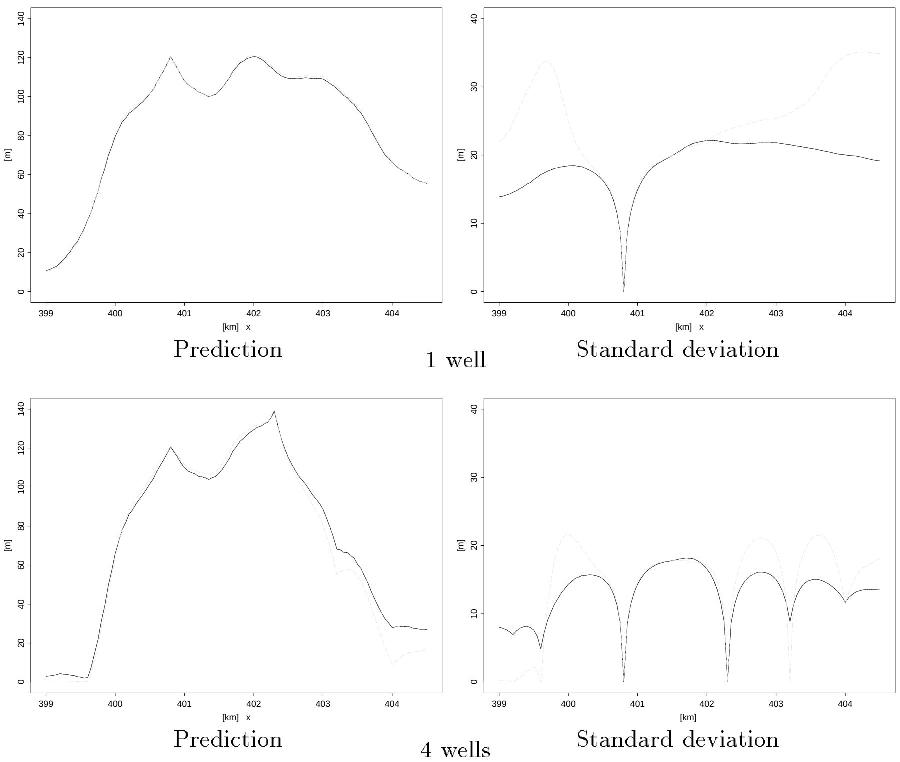


Figure 4. Cross section of reservoir thickness above $z_c = 4000$. —— prediction and standard deviation related to $\hat{V}^D(\cdot)$; - - - prediction and standard deviation related to $\hat{V}^G(\cdot)$.

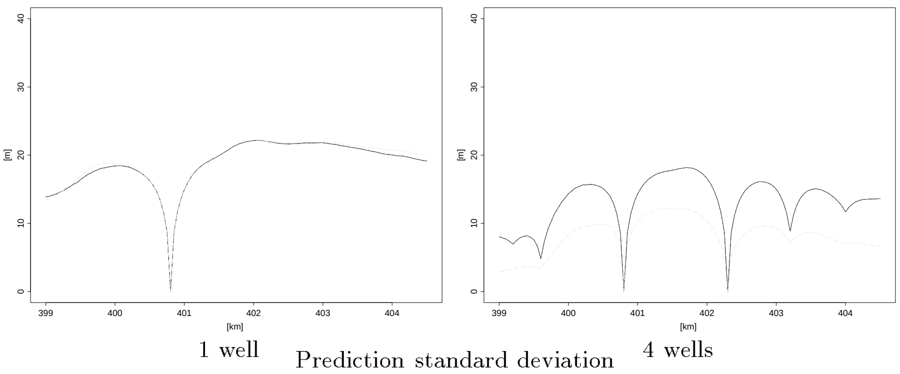


Figure 5. Cross section of reservoir thickness above $z_c = 4000$. —— standard deviation related to $\hat{V}^D(\cdot)$ - - - standard deviation related to $\hat{V}^G(\cdot)$.