PETROLEUM GEOSTATISTICS

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Abstract. For reservoir evaluation, general reservoir information and reservoir-specific observations are available. The importance of establishing a prior model for the reservoir characteristics which reflects the overall uncertainty is emphasized. Moreover, likelihood functions which represent the data collection procedures are formally defined. The integration of the information is done in a Bayesian framework. The requirement that sampling from the posterior must be possible put serious constraints on the model formulation. These constraints are thoroughly discussed.

1. Introduction

The primary objective of petroleum reservoir evaluation is to optimize reservoir management for the reservoir under study and to forecast the corresponding future production of oil and gas. The procedure for providing forecasts consists of two steps. First, establish a representation of relevant reservoir characteristics based on available information. Second, simulate the petroleum production from the reservoir by numerical simulation of fluid flow through this representation of the reservoir characteristics given the recovery strategy. In order to assess the uncertainty in the production forecasts, a stochastic model for the reservoir characteristics is defined. The predicted uncertainty is then obtained by Monte Carlo sampling from this model and thereafter simulation of fluid flow; see Lia et al (1996). The focus of this article is on stochastic modeling of the reservoir characteristics and the sampling techniques available.

The stochastic model for characteristics of the reservoir under study must be based on two types of information: (i) general reservoir information and (ii) reservoir-specific observations. The former consists of geological and petrophysical process understanding and observations in geological analogues and comparable reservoirs. The latter consists of observations in the reservoir under study as measurements in wells, data from seismic surveys and previous production history. Note that the former is experience-based while the latter are observations in the actual reservoir. The challenge is to combine these two types of information in an optimal manner for production forecasts and from a statistical point of view a Bayesian approach seems reasonable; see Box and Tiao (1973).

Heterogeneity modeling along the lines of Haldorsen and Lake (1984) concerns bias correction of the production forecasts. A large number of publications have been presented on this topic the last decade; see Haldorsen and Damsleth (1990), Omre (1991) and Dubrule (1993) for good overviews. Assessment of uncertainty in production forecasts has only recently been thoroughly discussed in the petroleum literature; see Omre et al (1993a) and Journel (1994). The objective of this article is to contribute to formalization of uncertainty assessments in production forecasts through formalizing the stochastic modeling of reservoir characteristics.

This article contains a discussion and formalization of how the general reservoir information and reservoir-specific observations should be combined. This is done in a Bayesian framework. It is required that sampling can be performed from the resulting stochastic model and this puts constraints on the complexity of the model. These constraints are thoroughly discussed. References to several examples are given.

2. Model Formulation

Consider a particular petroleum reservoir under study. The final aim is usually to optimize the reservoir management and to forecast the corresponding production characteristics. These characteristics may be cumulative production of oil and gas, water cuts, pressure changes etc and are denoted $q_p(t)$, where p and t represent a given recovery strategy and time, respectively. In the idealistic case exact forecast of production characteristics can be obtained through $q_p(t) = w(r,p)$, where $w(\cdot,\cdot)$ is the perfect model for fluid flow and r is the exact initial reservoir characteristics. Optimal reservoir management can be determined by choosing the strategy p which maximize production according to some criterion.

In practice, the initial reservoir characteristics are largely unknown at the stage of evaluation. Hence, it is convenient to define them to be stochastic and denoted by R. Which variables to include in R is defined by the input requirements from the fluid flow simulator, $w(\cdot,\cdot)$. The random field, R, is usually multivariate to represent characteristics like porosity, horizontal and vertical permeability, initial oil and gas saturations, fluid properties etc. Moreover, it must be spatial in order to represent the spatial variability

$$
\begin{array}{ccc}
o_w & o_s & o_p \\
\backslash & | & / \\
f(o_w|r) & f(o_s|r) & f(o_p|r) \\
= \int f(o_w|r,\psi_w)f(\psi_w)\mathrm{d}\psi_w & = \int f(o_s|r,\psi_s)f(\psi_s)\mathrm{d}\psi_s & = \int f(o_p|r,\psi_p)f(\psi_p)\mathrm{d}\psi_p \\
\backslash & | & /
\end{array}
$$

$$\boxed{f(r) = \int_{\Omega_\Theta} f(r|\theta) \cdot f(\theta) d\theta}$$

$$|$$

$$f(r|o) = \text{const} \cdot f(o_w|r) f(o_s|r) f(o_p|r) f(r)$$

$$[\ f(r, \theta, \psi|o) = \text{const} \cdot f(o_w|r, \psi_w) f(o_s|r, \psi_s) f(o_p|r, \psi_p) f(\psi) f(r|\theta) f(\theta)\ ]$$

Figure 1. Outline of model used to obtain the posterior distribution

of the characteristics mentioned above. The associated probability density function, pdf, is termed $f(r)$. To simplify the discussion and because the focus is on reservoir uncertainty, the fluid flow model is assumed to be perfect and represented by a reservoir production simulator. This is a severe assumption that should be relaxed in later work, of course.

Since the initial reservoir characteristics are defined to be stochastic, the production characteristics become stochastic as well and are represented by $Q_p(t) = w(R, p)$. The associated pdf, $f(q_p)$, is defined by the pdf $f(r)$ and the function $w(\cdot, \cdot)$. The latter constitutes a set of stochastic differential equations, see Holden and Holden (1991), which cannot be solved analytically for realistic models. Hence, the uncertainty, or the pdf, of the production characteristics has to be assessed by sampling through a Monte Carlo approach. This entails firstly to sample from $f(r)$ and thereafter to determine a sample from $f(q_p)$ through $w(\cdot, \cdot)$. Clever sampling designs can of course be imagined; see Damsleth et al (1992). Conditional forecast of production given the reservoir-specific observations is obtained in a similar manner through the conditional stochastic model for the reservoir characteristics. The focus of this paper is on the latter.

The available information consists of general reservoir information and reservoir-specific observations, listed above as (i) and (ii), respectively. Based on the former, a prior stochastic model for the reservoir characteristics, R, must be established. The model must be conditioned on the actual reservoir-specific observations made. Hence, the goal of this study is to obtain properties of the posterior stochastic model for the reservoir characteristics conditioned on the available observations, either analytically or at least by sampling. An outline for obtaining this posterior stochastic model is given in Figure 1.

The reservoir-specific observations are of three types: observations in wells, denoted by O_w; seismic data, denoted by O_s; and production tests and history, denoted by O_p. Denote them by $O = (O_w, O_s, O_p)$. For the reservoir under study, let the observed values be o_w, o_s and o_p, respectively. In order to link these reservoir-specific observations to the reservoir characteristics, R, the acquisition procedures must be modeled through likelihood functions. For well observations, the likelihood function, $f(o_w|r, \psi_w)$, specifies the pdf of observing o_w given the true reservoir characteristics, r, and the some data acquisition parameters ψ_w. The latter are usually unknown parameters in the measurement equipment. The associated conditional stochastic variable can be written as $O_w = g_w(r, \psi_w) + U_w$, where $g_w(r, \psi_w)$ is the expected response in the well data acquisition equipment to the reservoir characteristics r and parameters ψ_w. U_w is some stochastic error term. The function $g_w(r, \psi_w)$ is often termed the transfer function and is a subject of intensive research among petrophysicists. Correspondingly, $f(o_s|r, \psi_s)$ and $f(o_p|r, \psi_p)$ are likelihood functions for seismic data and production test and history, respectively. The associated $g_s(r, \psi_s)$ for seismic data can be derived from wave equations along the lines adopted in the geophysics community and ψ_s may be wavelet parameters. The $g_p(r, \psi_p)$ is defined from Darcy's law and may be represented by a fluid flow simulator. Note that the formalism allows a trade-off between refinement in the mathematical modeling of $g_w(\cdot,\cdot)$, $g_s(\cdot,\cdot)$ and $g_p(\cdot,\cdot)$ and the size of the variance in the respective error terms U_w, U_s and U_p. If rough models for the former is used with associated larger variance in the latter, the reservoir-specific observations, o_w, o_s and o_p, are less appreciated in the evaluation.

The data acquisition parameters, termed $\psi = (\psi_w, \psi_s, \psi_p)$, are of course also unknown and hence specified to be stochastic, denoted by Ψ. Experience with the measurement equipment make it possible to assign a prior pdf, $f(\psi) = f(\psi_w) \cdot f(\psi_s) \cdot f(\psi_p)$, to those parameters. It is natural to assume parameters for the various sources of information to be independent, at least a priori. Hence, the likelihood function for observing the reservoir-specific observations given the reservoir characteristics is

$$f(o|r) = \int_{\Omega_\Psi} f(o|r, \psi) \cdot f(\psi) \mathrm{d}\psi, \tag{1}$$

where Ω_Ψ is the sample space of the data acquisition parameters ψ.

The available reservoir-specific observations are associated with the reservoir characteristics, R, through the likelihood functions, see Figure 1. Hence, to determine R appears as an inverse problem. Unfortunately, the reservoir characteristics, R, are of very high dimension due to its multivariate, spatial nature and the problem appears ill-posed. This entails that R cannot be uniquely determined by the reservoir-specific observations, o_w, o_s and o_p. The statistical solution to this involves assigning a prior stochastic model to the reservoir characteristics.

The prior stochastic model for R is represented by a pdf $f(r)$. It must be assessed from general reservoir information consisting of geological process understanding, observations from analogues and experience from comparable reservoirs. Hence, it is based on the prior conceptions of the geoscientists before measurements in the reservoir under study are made. It is crucial for the evaluation that $f(r)$ gives a realistic description of the prior uncertainty about R. This is most easily obtained by using a hierarchical model for $f(r)$, i.e. to let the distribution for R depend on a set of model parameters θ, which are themselves considered as stochastic with an associated prior pdf, $f(\theta)$. In order to make the assessment of $f(\theta)$ easier, it is important to parameterize the distribution for R so that θ is easily interpretable, for example as expected values for poro-perm characteristics, net-to-gross values and expected sizes of facies objects. The θ may also be used to combine models for different geological interpretations for the reservoir under study. The prior distribution for θ can then, for example, be assessed by requesting geoscientists to identify analogues and other reservoirs that are comparable to the reservoir of interest and to utilize experience and observations from these to determine $f(\theta)$. Based on this, the following relation is defined

$$f(r) = \int_{\Omega_\Theta} f(r|\theta) \cdot f(\theta) \mathrm{d}\theta, \tag{2}$$

where Ω_Θ is the sample space of the model parameters Θ.

The conditional pdf $f(r|\theta)$ represents the uncertainty in the reservoir characteristics for given values of the model parameters. This corresponds to the geometrical rearrangements of characteristics and can be termed heterogeneity uncertainty. The pdf $f(\theta)$ represents the prior uncertainty in the model parameters and can be termed model parameter uncertainty. Note that the former, heterogeneity uncertainty, corresponds to the heterogeneity evaluation approach as initiated by Haldorsen and Lake (1984). The latter, model parameter uncertainty, appears to be very important whenever assessment of uncertainty is the objective; see Omre et al (1993a) and Lia et al (1996).

With likelihood and prior as defined above, the posterior distribution for the reservoir characteristics is given by

$$\begin{aligned} f(r|o) &= \text{const} \cdot f(o|r) \cdot f(r) \qquad (3) \\ &= \text{const} \cdot \int_{\Omega_{\Psi_w}} f(o_w|r, \psi_w) \cdot f(\psi_w)\mathrm{d}\psi_w \cdot \int_{\Omega_{\Psi_s}} f(o_s|r, \psi_s) \cdot f(\psi_s)\mathrm{d}\psi_s \\ &\quad \cdot \int_{\Omega_{\Psi_p}} f(o_p|r, \psi_p) \cdot f(\psi_p)\mathrm{d}\psi_p \cdot \int_{\Omega_\Theta} f(r|\theta) \cdot f(\theta)\mathrm{d}\theta, \end{aligned}$$

by assuming O_w, O_s and O_p to be conditionally independent for given R and Ψ. Note that O_w, O_s and O_p are actually three blocks of observations from the well, seismic and production data acquisition tools, respectively. Each block contains multivariate observations that are highly dependent. The three blocks, O_w, O_s and O_p are based on different tools and procedures and hence, it is reasonable to consider them to be conditionally independent given the reservoir characteristics.

The posterior pdf of the reservoir characteristics, $f(r|o)$, is obtained from the stochastic models previously defined. Analytical treatment of the posterior distribution is feasible in very few cases only. However, sampling is possible from a fairly large class of models. But in the general case even sampling is totally intractable. This is the topic of the following sections.

The framework for stochastic reservoir evaluation defined in this section is along the lines of traditional statistical modeling. Ill-posed inverse problems are frequently observed and introduction of a prior to compensate for this is often done. Bayesian approaches are in frequent use whenever few observations and extensive expert experience are available. The framework is familiar but the problem of reservoir characterization has several special features. First of all, the target variable in reservoir characterization, R, is of extremely high dimension due to its multivariate and spatial nature. The amount of information in the reservoir observations normally vary considerably, spatially. Close to wells the observations almost uniquely define the reservoir characteristics but further away from wells the observations carry much less information. This makes specification of the prior very important. The likelihood functions, including the transfer functions, normally are very complex and require the solution of large sets of differential equations. Reliable numerical solutions to these require large computer resources. Luckily, considerable geoscientific knowledge is available about reservoir characteristics. Moreover, the reservoir is usually only a small unit in a larger regional geological structure and hence analogues can be identified. Once the posterior stochastic model for the reservoir characteristics is established, there is a need to sample from it in order to make further processing possible, rather than identifying an optimal reservoir characterization based on, for example, the maximum a posteriori criterion. Finally, reservoir evaluation is a dynamic task with frequent updates of the posterior stochastic model as more reservoir-specific observations become available through infill drilling, additional seismic surveys and current production history. The real challenge is to adapt the framework defined above so that all features mentioned are accounted for in a balanced manner.

3. Stochastic Simulation

The stochastic model of interest is the posterior pdf for the reservoir characteristics given the reservoir-specific observations, $f(r|o)$, specified in equation (3). For most models relevant in reservoir characterization, the integrals in this expression cannot be determined analytically and this makes direct sampling from the distribution problematic. But the problem can be avoided by instead consider the joint posterior pdf $f(r,\theta,\psi|o)$. This distribution is given by

$$f(r,\theta,\psi|o) = \text{const} f(o_w|r,\psi_w) f(o_s|r,\psi_s) f(o_p|r,\psi_p) f(\psi) f(r|\theta) f(\theta) \quad (4)$$

and contains no problematic integrals. Moreover, if (r,θ,ψ) is a sample from $f(r,\theta,\psi|o)$, r is obviously a sample from the marginal pdf $f(r|o)$ of interest.

Sampling from a pdf is most efficiently done whenever the pdf can be factorized into lower dimensional pdf's, preferable one-dimensional ones. If $T = (R,\Theta,\Psi) = (T^1,\dots,T^n)$ is a decomposition of the reservoir characteristics and associated model parameters into univariate random variables, the joint posterior distribution above can be expressed as

$$f(r,\theta,\psi|o) = f(t|o) = f(t^1|o)\cdot f(t^2|t^1,o)\cdot\ldots\cdot f(t^n|t^{n-1},\dots,t^1,o). \quad (5)$$

The sequential simulation algorithm of Gómez-Hernández and Journel (1993) or Omre et al (1993b), can then be used. But this requires that the following integrals can be determined analytically for $i = 2,\dots,n$

$$\int_{\Omega_{T^i}} \cdots \int_{\Omega_{T^n}} f(t|o) dt^{i+1}\cdot\ldots\cdot dt^n, \quad (6)$$

where Ω_{T^i} is the sample space of the random variable T^i. This is a serious constraint and can be dealt with only for very particular models, most commonly specific models from the Gaussian and Poisson families.

Sampling from complex stochastic models has in recent years been a subject of intensive research in the statistical community. For reviews of the theory and techniques, see Besag and Green (1993) and Neal (1993). Markov chain Monte Carlo techniques have been the central topic and the Metropolis-Hastings algorithm is the most prominent class of algorithms. All these algorithms have in common that they are iterative and that convergence, i.e. sampling from the correct stochastic model, is ensured in the limit only. The advantage is that sampling from relatively complex stochastic models can be performed. Note that the simulated annealing algorithm, in the geostatistical community frequently used for simulation (Deutsch, 1993; Hegstad et al, 1993), belongs to the class of Markov chain Monte Carlo techniques.

The Metropolis-Hastings algorithm is iterative and each iteration consists of two steps. Let $t = (r,\theta,\psi)$ denote the current state in an algorithm for sampling from $f(t|o)$. Each iteration then consists of first (i) drawing a potential new state $\tilde{t}$ according to a transition matrix $Q(t \to \tilde{t})$ and (ii) accepting $\tilde{t}$ as the new current state with probability

$$A(t \to \tilde{t}) = \min \left\{ 1, \frac{f(\tilde{t}|o)}{f(t|o)} \frac{Q(\tilde{t} \to t)}{Q(t \to \tilde{t})} \right\}, \tag{7}$$

otherwise t is kept as the current state. Only very weak restrictions exist on the choice of $Q(t \to \tilde{t})$ and this makes it a very flexible algorithm. An important property of the procedure is also that the pdf $f(t|o)$ need to be known up to a normalizing constant only because the constant cancels in the expression for the acceptance probability, $A(t \to \tilde{t})$. Hence, the posterior pdf must be of the form

$$f(r, \theta, \psi|o) = \text{const} \cdot h(r, \theta, \psi), \tag{8}$$

where $h(r, \theta, \psi)$ is a known function. This requires the normalizing constants in the pdf's $f(r|\theta)$, $f(o_w|r, \psi_w)$, $f(o_s|r, \psi_s)$ and $f(o_p|r, \psi_p)$ to be known because they are functions of the conditioning variables, θ, r and ψ. However, the fact that no similar restriction is necessary for the normalizing constant in the pdf for the reservoir model parameters, $f(\theta)$, still gives a lot of flexibility in the choice of prior stochastic model.

To determine the normalizing constant in $f(r|\theta)$ analytically is difficult in the general case, since integrating over the multivariate, spatial random variable r is usually non-trivial. Hence, this constitutes a real constraint. In practice, one has to assume that $f(r|\theta)$ belongs to the Gaussian or Poisson families of random fields with parameters θ. Note, however, that the pdf of the stochastic reservoir model parameters, Θ, can be chosen arbitrarily without constraints. This entails that the prior pdf for the reservoir characteristics, $f(r) = \int f(r|\theta) \cdot f(\theta) \mathrm{d}\theta$, still belongs to a fairly large class of pdf's. One could envisage to determine the normalizing constant in $f(r|\theta)$ by sampling-based techniques (Heikkinen and Högmander, 1994; Higdon et al, 1995) but this is computationally prohibitive unless θ is very low-dimensional.

To determine the normalizing constants in $f(o_w|r, \psi_w)$, $f(o_s|r, \psi_s)$ and $f(o_p|r, \psi_p)$ analytically is of course also impossible in the general case but with reasonable assumptions this can normally be done. If the observation error follows familiar parametric classes of pdf's, for example Gaussian, with parameters dependent on r and ψ, the normalizing constant is usually analytically available. But note that the assumptions, in the case of additive noise, do not involve the transfer functions, $g_w(r, \psi_w)$, $g_s(r, \psi_s)$ and $g_p(r, \psi_p)$. The prior pdf for the data acquisition parameters, $f(\psi) = f(\psi_w) \cdot f(\psi_s) \cdot f(\psi_p)$ can be chosen without constraints.

As previously mentioned, Markov chain Monte Carlo algorithms only give samples from the specified distribution in the limit. It therefore becomes essential to decide when a sufficiently good approximation is reached and much research has concentrated on this topic. One approach used is to look for theoretical bounds on the number of iterations necessary to reach within a given distance from the specified distribution (Meyn and Tweedie, 1994; Rosenthal, 1994) but so far no good bounds seem to exist for models of the complexity necessary in reservoir characterization. A technique frequently used also for complex models is output analysis; see Ripley (1981) and references therein. One approach is to plot important univariate characteristics of the realizations against number of iterations and to look for when they seem to have stabilized statistically. Alternatively, if some property of the pdf is analytically known, this can be utilized by comparing the theoretically known properties with corresponding estimated values.

4. Applications

The framework presented in the previous sections has two major components, the prior model for the reservoir characteristics, $f(r)$, and the likelihood functions for the reservoir-specific observations, $f(o|r)$. The former is assessed through general reservoir information, while the latter is assessed through knowledge of the data acquisition procedures for the reservoir-specific observations. One may consider the posterior model for the reservoir characteristics, $f(r|o)$, as an update of the prior model by the information in the reservoir-specific observations. Moreover, the Bayesian formalism allows for sequential updating as more observations are made available.

The framework presents a unified concept for prior models like Gaussian random functions, Markov random fields, marked point fields etc. The likelihood functions can represent almost any data acquisition procedure and provide a formalism for combining them. In particular, the object function in the annealing technique used in geostatistics can be represented as a likelihood function and annealing can be put in the framework defined above. Note also that the complete kriging theory can be put into the framework defined. However, for complex models one has to rely on Markov chain Monte Carlo sampling procedures and the problems in having them to converge should not be under-estimated.

The rest of this section contain brief presentations of some studies, in which this formalism is explicitly used. Other work has certainly also used the same line of thought, but very rarely is the formalism clearly exposed in articles belonging to the geostatistical flora.

In Eide et al (1996), integration of well observations and seismic data in reservoir characterization is discussed. The prior model for the characteristics is a Gaussian random function of the Bayesian kriging type; see Omre and Halvorsen (1989). The likelihood functions depend on one data acquisition parameter related to the seismic wavelet. Given this parameter, the observations are linear combinations of the reservoir characteristics with observation errors being additive Gaussian. If the wavelet parameter was known, the model would be fully Gaussian and subject to complete analytical treatment. Hence, the very efficient sequential simulation algorithm could be applied. Moreover, the best prediction of the reservoir characteristics under the criteria expectation of posterior pdf and maximum posterior pdf (MAP) coincide and is identical to the appropriate kriging predictor. This demonstrates that the kriging theory fits into the framework. In the case with unknown wavelet parameter to be simultaneously estimated, a Markov chain Monte Carlo procedure must be used as demonstrated in Eide et al (1996).

In Tjelmeland and Omre (1996), the classical sand/shale model of Haldorsen and Lake (1984) is extended. The prior model for shale locations is a hierarchical marked point field which allows clustering of shale units. Likelihood functions for both well observations, seismic data and production history is defined. The likelihood functions contain both non-linear relations of reservoir characteristics and non-additive observation errors. The model is completely intractable analytically and a Metropolis-Hastings algorithm is used to sample from the posterior model. In order to obtain convergence within reasonable time, a clever implementation is required; see Tjelmeland and Omre (1996). The paper demonstrates how complex marked point fields subject to complex conditioning can be sampled from.

In Syversveen and Omre (1996), the conditioning problem in Haldorsen and Lake (1984) is solved. The prior model for the shale units allows very complex geometries by using a hierarchical marked point field also including spatial interaction between units. The likelihood function represents exact observations of shale units in an arbitrary number of vertical wells. More wells may penetrate the same shale unit.

The model is too complex to allow full analytical treatment, but due to conditional independence of the geometries of individual shale units, partial analytical treatment is possible. The conditioning problem for individual shales can be solved analytically. The approach is demonstrated on real data from an outcrop.

In Hegstad and Omre (1996), conditioning of reservoir characteristics on production history, or history matching, is discussed. The prior model for the reservoir characteristics is a Gaussian random function of the Bayesian kriging type. The likelihood functions represent observations of the characteristics in wells and production data over some time. The latter requires a transfer function representing fluid flow, which is highly non-linear and very resource requiring to compute. The focus of the study is forecast of production.

The posterior model can only be assessed through a Markov chain Monte Carlo procedure which will be very resource requiring. History matching is a hard problem and further work on the efficiency of the sampling procedure is needed.

5. Closing Remarks

Stochastic modeling in reservoir characteristics is a hard problem. The variables of interest are defined in extremely high dimensions, many types of observations are associated to the variables through complex relations and considerable expert experience is available. Moreover, the models have to be sequentially updated whenever more observations are made available through further drilling, new seismics or production. A solution in a statistical spirit can be found in a Bayesian framework.

The model contains two major components, a prior model and likelihood functions. It is important that the prior model reflects the prior uncertainty, both inherent heterogeneity in the characteristics and the uncertainty concerning the reservoir model parameters. Moreover, uncertainty concerning geological interpretation of the reservoir can be included. The reservoir-specific observations are collected in various ways and it is important that they are appreciated according to their accuracy, precision and interdependence. The modeling of the data acquisition procedure in likelihood functions ensure this.

The posterior model of the reservoir characteristics usually is too complex to allow complete analytical treatment. Hence, one has to rely on sampling, normally based on Markov chain Monte Carlo procedures. The sampling will in many cases be very computer resource requiring and clever implementations are crucial. It is of utmost importance, however, to have an established framework for the solution since this makes it easier to utilize increased computer power when it becomes available.

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