Stochastic simulation of porosity and acoustic impedance conditioned to seismic data and well data

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Summary

Stochastic modeling of the acoustic impedance and porosity conditioned to seismic and well data can be done using the linear relationship between reflection coefficients and seismic amplitude under a convolution model.

Introduction

Stochastic modeling is frequently used in reservoir characterization to integrate various types of information and to quantify uncertainty. A Bayesian framework for stochastic reservoir characterization is described in Lia, Omre, Tjelmeland, Holden and Egeland (1997) and Omre and Tjelmeland (1997). In Eide, Omre and Ursin (1997) this approach is applied to the problem of seismic inversion under a simplified linear model. In this paper the approach is generalized to cover nonlinear relations between the acoustic impedance and the reflection coefficients.

Integration of seismic data is particularly important since it gives good spatial coverage of the reservoir. Acoustic impedances have been included trough cokriging (Doyen 1988) and indicator models (Zhu and Journel 1993). These methods assume that acoustic impedance is available from conventional inversion methods, hence the uncertainty in the inversion process and the ambiguity of the seismic data are not properly accounted for. Bortoli, Alabert, Haas and Journel (1993) and Haas and Dubrule (1994) suggest using stochastic models at an earlier stage in the inversion process. To account for the nonlinear relationship between seismic amplitude and acoustic impedance, ad hoc sampling techniques are used. Abrahamsen, Buland, Bølviken, Hauge, Hektoen and Skorstad (1996) use a more formal approach and improve the efficiency of the sampling methods by utilizing a linearized model.

In our model, the reflection coefficients are linearly related to seismic amplitude under a convolution model. Under a Gaussian based model, the stochastic model for the reflection coefficients conditioned to seismic amplitude data can be obtained analytically. Realizations from this conditional model of the reflection coefficients can then be nonlinearly transformed to give acoustic impedance realizations. Finally, porosity realizations conditioned to seismic amplitude data are obtained via the acoustic impedance realizations. Observations of porosity and acoustic impedance in wells are also included.

Bayesian framework

Let the reservoir characteristics of interest, like porosity, acoustic impedance, permeability etc, be denoted r. Due to the uncertainty and lack of detailed knowledge about the reservoir, it is convenient to represent it as a stochastic variable R. General geological knowledge about feasible values of r provides a prior probability density function $f(r)$. This represents the knowledge about the reservoir before any reservoir specific observations are included.

Reservoir specific observations, like seismic data, well logs, production tests etc, will be denoted o with a stochastic counterpart O. The link between reservoir characteristics and observations can be modeled as

$$O = g(R) + U, \tag{1}$$

where $g(R)$ is a forward transfer function, for example a seismic forward model, and U represents measurement error and modeling error. The link between reservoir characteristics and observations is represented by an observation likelihood function $f(o|r)$, interpreted as the probability of observing o given the reservoir r.

The objective of the study is to determine the posterior probability of the reservoir characteristics given the reservoir specific observations. It can be represented as

$$f(r|o) = \text{const} \times f(o|r) f(r). \tag{2}$$

Conditioning porosity and acoustic impedance to seismic data and well data

Let the reservoir characteristics be $R = (\Phi, Z)$ where Φ is porosity and Z is acoustic impedance. The reservoir specific observations $O = (D, \Phi^o, Z^o)$ are seismic amplitude data D covering the complete reservoir and well observations Φ^o and Z^o of porosity and acoustic impedance respectively along the well trajectories.

The relationship between these stochastic variables is illustrated in Fig. 1 and will be explained below. Seismic amplitude data are modeled as a convolution between a seismic wavelet and a sequence of reflection coefficients $C(x_i, t_i)$ with x_i and t_i being the location and time-reference. The reflection coefficients are given as

$$C(x_i, t_i) = \frac{Z(x_i, t_{i+1}) - Z(x_i, t_i)}{Z(x_i, t_{i+1}) + Z(x_i, t_i)} \tag{3}$$

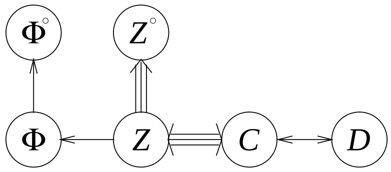

Figure 1: Graphical model showing relations between variables involved in the reservoir model. Seismic data D and reflection coefficients C have a bi-Gaussian distribution. Between C and the acoustic impedance Z there is a deterministic relation. Porosity Φ is Gaussian given Z. Well observations of acoustic impedance, Z^o, are a subset of Z. Well observations of porosity, Φ^o, can be any linear function of Φ.

We assume that $C(\cdot,\cdot)$ can be described as a Gaussian random field with zero mean and covariance function

$$\mathrm{Cov}(C(x_i,t_i),C(x_j,t_j)) = \sigma^2\rho(x_j-x_i,t_j-t_i) \qquad (4)$$

where σ^2 is the variance and $\rho(\cdot,\cdot)$ the spatial correlation function of $C(x,t)$. This defines the prior model of C.

The convolution between reflection coefficients and the seismic wavelet can be represented by the linear forward model

$$[D|C=c] = Ac + U_D \qquad (5)$$

where D is seismic amplitude discretized on a grid of (x,t)-values, A is a matrix defined by the seismic pulse, C is a vector containing reflection coefficients in all location defined by the grid, and the error term U_D has a zero-mean Gaussian distribution with covariance matrix Σ_D. The left hand term represents the stochastic variable D given that the values of C are c. Then (D,C) is bi-Gaussian, which is represented by a two-sided arrow in Fig. 1. It is easy to show that the posterior distribution of C given seismic data $D=d^o$ is a Gaussian distribution with

$$\mathrm{E}(C|D=d^o) = \Sigma_C A^T(A\Sigma_C A^T+\Sigma_D)^{-1}d^o \qquad (6)$$
$$\mathrm{Cov}(C|D=d^o) = \Sigma_C - \Sigma_C A^T(A\Sigma_C A^T+\Sigma_D)^{-1}A\Sigma_C \qquad (7)$$

where Σ_C is the covariance matrix of C, with entries of Σ_C being defined by the covariance function (4). The posterior distribution of C given seismic data and well observations of C has a similar formula.

Acoustic impedance can be calculated from reflection coefficents by the recursive formula

$$Z(x_i,t_{i+1}) = \frac{1+C(x_i,t_i)}{1-C(x_i,t_i)}Z(x_i,t_i). \qquad (8)$$

Given $Z(x_i,t_0)$ this is a nonlinear, one-to-one, deterministic relation between C and Z, represented by a triple two-sided arrow in Fig. 1.

Several studies have shown empirically a linear dependence between porosity Φ and acoustic impedance Z within one facies, see for example Rasmussen and Maver (1996). A model that satisfies these requirements, is the following

$$[\Phi(x_i,t_i)|Z(x_i,t_i)=z_i] = \mu_\Phi(z_i)+U_\Phi \qquad (9)$$

where $\mu_\Phi(z_i)$ is a known function of z_i and U_Φ is Gaussian. Then Φ given Z is a Gaussian random field. This relation is represented by the one-sided arrow from Z to Φ in Fig. 1.

There is no direct relation between C and Φ, the relation is only through Z as indicated in Fig. 1. This is in accordance with empirical results showing no correlation between porosity and reflection coefficients, and theory of Gaussian random fields if C is defined as the derivative of Z, see eq. (3).

The posterior distribution of the reservoir characteristics R, being porosity Φ and acoustic impedance Z; given the reservoir specific observations O, being seismic data D and well observations Φ^o and Z^o of porosity and acoustic impedance respectively, can be factorized

$$f(r|o) = f(\phi,z|\phi^o,z^o,d) = f(\phi|z,\phi^o,z^o,d)\cdot f(z|\phi^o,z^o,d). \qquad (10)$$

The graphical model in Fig. 1 shows that Φ and D are conditionally independent given Z. This simplifies the first factor on the right-hand side of eq. (10). The second factor can be simplified if porosity well observations and acoustic impedance well observations are from the same spatial locations. Assume that z^o are exact observations, so that z^o is a subset of z (triple one-sided arrow in Fig. 1), and that the relation between porosity and acoustic impedance is as in eq. (9). Then porosity observations ϕ^o give no extra information on the acoustic impedance and

$$f(\phi,z|\phi^o,z^o,d) = f(\phi|z,\phi^o)\cdot f(z|z^o,d). \qquad (11)$$

This shows that samples from $f(\phi,z|\phi^o,z^o,d)$ can be obtained via sequential sampling from $f(z|z^o,d)$ and $f(\phi|z,\phi^o)$. The latter is Gaussian and straightforward to sample. Samples from $f(z|z^o,d)$ can be obtained by sampling $f(c|c^o,d)$, which is also Gaussian and straightforward to sample. Here c^o denotes reflection coefficients obtained from z^o through the one-to-one relation between C and Z. Realizations of C are then transformed to give acoustic impedance realizations Z through the same relation.

Example

The method described above will now be illustrated on a synthetic example where the true porosity and acoustic impedance are as shown in Fig. 2. This vertical cross section is discretized on a 50×14 grid with vertical grid resolution 1 ms. The true impedance cross section was generated from a Gaussian random field with constant

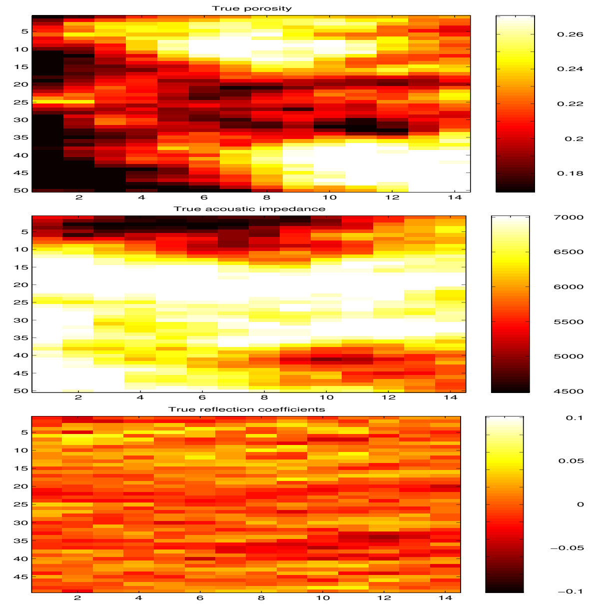

Figure 2: True porosity (upper), acoustic impedance (middle) and reflection coefficients (lower).

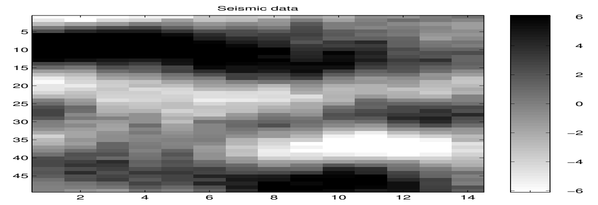

Figure 3: Seismic data generated by convolution of true reflection coefficients with 40 Hz Ricker wavelet and addition of white noise.

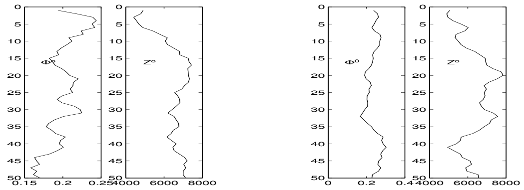

Figure 4: Porosity and acoustic impedance observations from wells at lateral position $x = 3$ (left) and x=10 (right).

expectation 5600, variance 1000^2 and spatial correlation function $\rho(\Delta) = \exp(-3(\Delta/a)^\nu)$ where Δ is a distance measure, range $a = 20$ and $\nu = 1.5$. The true porosity cross section was generated using the model in eq. (9) with $\mu_\Phi(z_i) = 0.464 - 4 \cdot 10^{-5} z_i$, and U_Φ a Gaussian random field with variance 0.03^2 and the same correlation function as Z. The a priori covariance function of C is taken to be the covariance function of $[Z(x_i, t_{i+1}) - Z(x_i, t_i)]/(2\theta)$ where θ is the expected value of Z.

Seismic amplitude data (Fig. 3) have been generated by convolving a 40 Hz Ricker wavelet with the reflection coefficients calculated from the true acoustic impedances (Fig. 2). White noise with variance 0.25 has been added to the seismic amplitude data. There are two wells at lateral position $x = 3$ and $x = 10$ with porosity and acoustic impedance observations (Fig. 4). Well observations are assumed to be exact.

Fig. 5 shows three realizations from the posterior distribution of reflection coefficients given seismic and well data. These are back-transformed to acoustic impedance realizations shown in Fig. 6 which is used for simulation of porosity (Fig. 7) as explained in the previous section. The true values for $z_0 = Z(x_i, t_0)$ have been used in the back-transformation here. Alternatively, samples from $f(z_0|c, z^o)$ could be used. The figures show that the large scale structures of porosity and acoustic impedance are reproduced, but there is considerable variation in the details, reflecting the uncertainty in the model. Remember that the a priori model is completely stationary, so that all similarity between the three realizations comes from the conditioning data. Note the high porosity area in the bottom right corner in Fig. 7. This shows the impact of the seismic data since the high porosity area clearly persists away from the well at $x = 10$.

Conclusions

Under a Gaussian based model the posterior distribution for reflection coefficients conditioned to seismic data and well observations can be obtained analytically. A nonlinear, deterministic transformation takes reflection coefficients into acoustic impedances that can be used for porosity modeling conditioned to well observations of porosity. This approach incorporates uncertainty in the inversion process and avoids the sampling problems that may occur if acoustic impedances are to be sampled directly.

Acknowledgments

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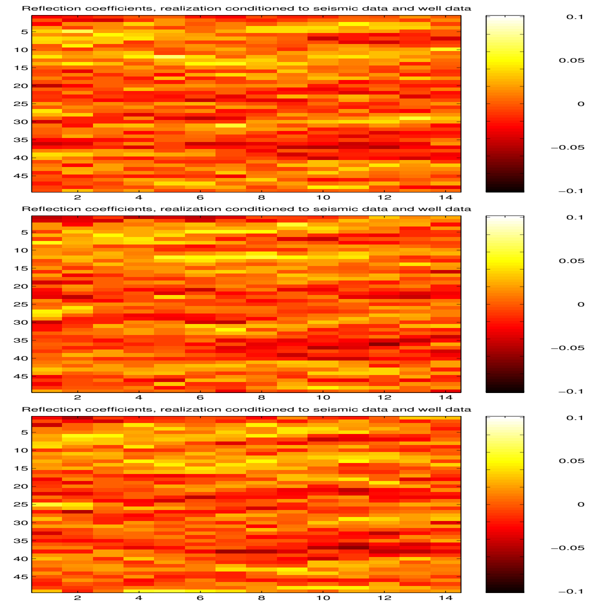

Figure 5: Three realizations from the posterior distribution of reflection coefficients conditioned to seismic and well data.

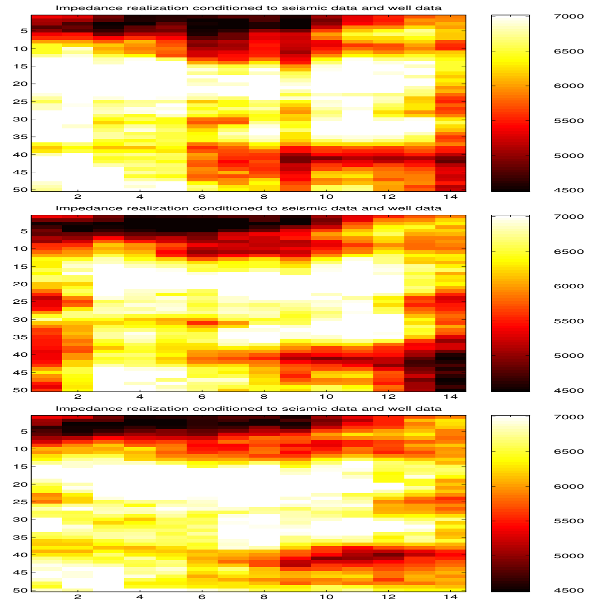

Figure 6: Three realizations from the posterior distribution of acoustic impedance conditioned to seismic and well data.

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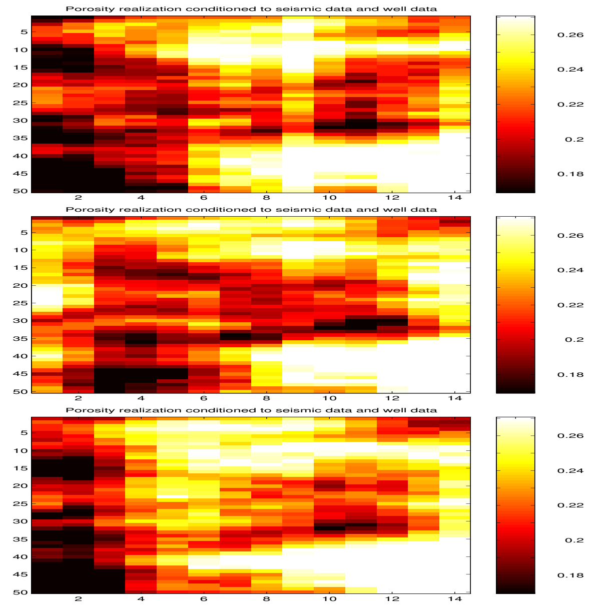

Figure 7: Three realizations from the posterior distribution of porosity conditioned to seismic and well data.