Repeat and Mirror-Image Patterns and Correlation Structures of Plackett-Burman Designs, their Foldovers and Half-Fractions

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November 2, 1998

1 Introduction

One of the rationales for using 2^{k-p} fractional factorial designs (Finney, 1945; Box and Hunter, 1961) in screening situations is their projection properties. When projected into a few columns these designs may obtain better resolution. For example will a 2_{IV}^{8-4} fractional factorial design for studying 8 factors in 16 runs project into a replicated 2^3 full factorial design for any three factors. Thus, if only a few of the factors considered initially are active, often referred to as *factor sparsity*, these designs are regarded well suited for an efficient screening of many factors in relatively few runs.

Plackett and Burman (1946) provided other screening designs that, assuming negligible interactions, enable estimation of all main effects with maximum precision for given number of runs. Plackett-Burman (PB) designs exist when the number of runs is a multiple of four. When the number of runs is a power of two they coincide with the 2^{k-p} fractional factorial (FF) designs, and are often called *geometric* designs. Otherwise they are called *non-geometric* designs. Computer search by Lin and Draper (1992) and Box and Bisgaard (1993) showed that some of the saturated non-geometric PB designs have better projection properties than the saturated FF designs. This rather surprising fact has lead to increased interest for the non-geometric PB designs with respect to their properties and ability to identify the active factors, and methods for analysing experiments from these designs. Results by Hamada and Wu (1992), Wang and Wu (1995) and Tyssedal and Samset (1997) show that the non-geometric PB designs are important screening designs even when interactions are present.

Having generated the *repeat and mirror-image patterns* (Draper and Lin, 1995) and the *correlation structures* for most of the PB designs, their foldovers and half-fractions, we found it useful to collect these results in a separate report. For projections into high dimensions the number of different patterns can be rather large. Thus, in order to keep the report as short as possible we have only focused on projections into four, five and three dimensions for the PB designs, their foldovers and half-fractions, respectively.

The repeat and mirror-image patterns of a design may provide useful information regarding the projection properties of the design. In fact these patterns show that the 68 run PB design is of projectivity $P = 4$. Hence it follows that the half-fraction of this design, which enables screening of 66 factors in 34 runs (Lin, 1993), is of projectivity $P = 3$. Also the 72, 80 and 84 run PB designs are of projectivity $P = 4$.

2 About Construction of the Designs and Notation

2.1 Construction of the Designs

In this report the columns or subsets of columns from the different designs are referred to by the number of which they are ordered in the designs. Hence the construction of the designs is of importance.

- For the cyclic designs the generators given by Plackett and Burman (1946) are used as the $(n-1)$ first rows in the first column. The other columns are found by cycling downwards and finally adding a last row of minus signs. Table 1 shows the 12 run PB design.

Table 1: The 12 run Plackett-Burman design.

run	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
1	+	-	+	-	-	-	+	+	+	-	+
2	+	+	-	+	-	-	-	+	+	+	-
3	-	+	+	-	+	-	-	-	+	+	+
4	+	-	+	+	-	+	-	-	-	+	+
5	+	+	-	+	+	-	+	-	-	-	+
6	+	+	+	-	+	+	-	+	-	-	-
7	-	+	+	+	-	+	+	-	+	-	-
8	-	-	+	+	+	-	+	+	-	+	-
9	-	-	-	+	+	+	-	+	+	-	+
10	+	-	-	-	+	+	+	-	+	+	-
11	-	+	-	-	-	+	+	+	-	+	+
12	-	-	-	-	-	-	-	-	-	-	-

- For the block-cyclic designs the blocks are cycled to the right; e.g. for the 28 run PB design the three blocks are arranged as

$$\begin{array}{ccc} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C} & \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{A} \end{array}$$

- Design are doubled according to

$$\begin{array}{ccc} \boldsymbol{A} & \boldsymbol{A} & \mathbf{1} \\ \boldsymbol{A} & -\boldsymbol{A} & -\mathbf{1} \end{array}$$

  Note that the column $(\mathbf{1}', -\mathbf{1}')'$ is not added when generating the 40, 56, 88 and 96 run PB designs by doubling.

- Foldover designs are generated according to

$$\begin{array}{cc} \boldsymbol{A} & \mathbf{1} \\ -\boldsymbol{A} & -\mathbf{1} \end{array}$$

- All half-fractions are found by using the last column as branching column, and then selecting all runs having (+) in the branching column.

2.2 Repeat and Mirror-image Structure

Draper and Lin (1995) characterise projected designs using repeat and mirror-image patterns. Consider the three first columns of the 12 run PB design given in Table 1. Here run 1 and 4 are identical to each other, i.e. run 1 and 4 are repeated runs. Furthermore run 11 is similar to these two but with all signs changed, i.e. run 11 is a mirror-image to run 1 and 4. This structure is by Draper and Lin (1995) written as [2 1] using a slightly different notation[1]. For the remaining runs we see that the structures for run (2, 5 and 8), (3, 7 and 10) and (6, 9 and 12) also is given by [2 1]. Thus, the repeat and mirror-image pattern for the first three columns can then be written as 4[2 1]. Examining all possible $\binom{11}{3} = 165$ projections into three columns shows that all these projections have the same repeat and mirror-image pattern 4[2 1].

These patterns are useful with respect to the projection properties of the designs. First, it is easy to see how many unique runs a given projection has. For the 12 run PB design every projection into three columns has the pattern 4[2 1] which implies 4(1+1)=8 unique runs. Since each projection into three columns has 8 unique runs, and thus contains a full 2^3, the 12 run PB designs is of projectivity $P = 3$. Secondly, for the 12 run PB design every projection into four columns has the pattern 1[2 0] 3[1 1] 4[1 0]. Thus, there are at least one run from 1+3+4=8 of the 8 possible repeat and mirror-image pairs. If the design is folded we are guaranteed that all projections into four columns have 16 of the possible 16 unique runs. Hence, the foldover design with 12 columns in 24 runs has projectivity $P = 4$.

2.3 Correlation Structure

The properties of the FF designs are often summarised through their alias-structure. These designs have the rather appealing property that all contrast columns, i.e. contrast columns corresponding to both main effect and interactions, are either orthogonal or fully confounded. This results in a rather simple alias-pattern. Also for the PB designs all contrasts columns corresponding to main effects are mutually orthogonal. However, contrasts columns corresponding to main effects can be correlated with contrast columns corresponding to interactions. The individual effects can be partially confounded resulting in a rather complex alias-pattern. For the 12 run PB design every main effect contrast is partially confounded with the 45 two-factor interactions not containing the main effect factor. For these designs it therefore might be helpful to look at the correlation structures between the different contrast columns. The (absolute value) correlation between two contrast columns x_i and x_j is here given by

$$|\frac{1}{n}\sum_k x_{ik}x_{jk}|.$$

2.4 Selecting a Subset of Columns with Higher Projectivity

Assume that a given design is of projectivity P. An interesting question is then how many of the columns from the given design that can be selected to give a projectivity $P + 1$ design. Instead of only looking at projectivity we rather list all columns that have at least a given number of unique runs when projected into $P + 1$ dimensions.

[1]Draper and Lin (1995) use the notation $[{}^{2}_{1}]^4$ instead of the notation 4[2 1] adopted here

3 Plackett-Burman Designs

3.1 The 8 Run Plackett-Burman Design

Table 2: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[2 2]	4	21
1 2 3	4[1 1]	8	28
1 2 4	4[2 0]	4	7
1 2 3 4	8[1 0]	8	28
1 2 3 6	4[1 1]	8	7
1 2 3 4 5	8[1 0]	8	21

Table 3: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	1.00
main	100.00	
main and 2fi	85.71	14.29
2fi	90.00	10.00
main and 3fi	88.57	11.43
2fi and 3fi	88.57	11.43
3fi	89.41	10.59

3.2 The 12 Run Plackett-Burman Design

Table 4: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[3 3]	4	55
1 2 3	4[2 1]	8	165
1 2 3 4	1[2 0] 3[1 1] 4[1 0]	11	330
1 2 3 4 5	1[1 1] 10[1 0]	12	396
1 2 3 4 6	1[2 0] 10[1 0]	11	66
1 2 3 4 5 6	12[1 0]	12	396
1 2 3 4 5 10	1[1 1] 10[1 0]	12	66
1 2 3 4 5 6 7	12[1 0]	12	330

Table 5: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$		
Between	0.00	0.33	0.67
main	100.00		
main and 2fi	18.18	81.82	
2fi	33.33	66.67	
main and 3fi	27.27	72.73	
2fi and 3fi	49.09	43.64	7.27
3fi	43.90	51.22	4.88

3.3 The 16 Run Plackett-Burman Design

Table 6: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[4 4]	4	105
1 2 3	4[2 2]	8	420
1 2 5	4[4 0]	4	35
1 2 3 4	8[1 1]	16	840
1 2 3 5	8[2 0]	8	420
1 2 3 11	4[2 2]	8	105
1 2 3 4 5	16[1 0]	16	1848
1 2 3 4 8	8[1 1]	16	840
1 2 3 5 6	8[2 0]	8	315

Table 7: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	1.00
main	100.00	
main and 2fi	93.33	6.67
2fi	94.23	5.77
main and 3fi	93.85	6.15
2fi and 3fi	93.85	6.15
3fi	93.93	6.07

Table 8: Unique runs when projected into 3 dimensions.

n	Columns
8	1:4 8 11 12 14
4	1:15

3.4 The 20 Run Plackett-Burman Design

Table 9: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[5 5]	4	171
1 2 3	4[3 2]	8	912
1 2 13	4[4 1]	8	57
1 2 3 4	4[2 1] 1[2 0] 3[1 1]	15	2736
1 2 3 7	1[3 1] 3[2 2] 4[1 0]	12	228
1 2 3 13	1[3 0] 3[2 1] 3[2 0] 1[1 1]	12	912
1 2 3 4 5	5[1 1] 10[1 0]	20	1881
1 2 3 4 6	2[2 0] 3[1 1] 10[1 0]	18	3078
1 2 3 4 7	1[2 1] 2[2 0] 4[1 1] 5[1 0]	17	684
1 2 3 4 8	1[2 1] 1[2 0] 5[1 1] 5[1 0]	18	1539
1 2 3 4 10	1[2 0] 4[1 1] 10[1 0]	19	1368
1 2 3 4 13	1[2 1] 3[2 0] 3[1 1] 5[1 0]	16	1026
1 2 3 4 17	3[2 0] 2[1 1] 10[1 0]	17	1368
1 2 3 7 13	1[2 1] 5[2 0] 1[1 1] 5[1 0]	14	171
1 2 3 8 17	4[2 0] 1[1 1] 10[1 0]	16	513

Table 10: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$			
Between	0.00	0.20	0.40	0.60
main	100.00			
main and 2fi	10.53	84.21		5.26
2fi	20.00	75.29		4.71
main and 3fi	15.79	79.26		4.95
2fi and 3fi	51.29	26.42	20.64	1.65
3fi	45.79	35.00	17.02	2.19

Table 11: Unique runs when projected into 4 dimensions.

n	Columns
16	None
15	1:5
12	1:19

3.5 The 24 Run Plackett-Burman Design

Table 12: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[6 6]	4	253
1 2 3	4[3 3]	8	1012
1 2 5	4[4 2]	8	759
1 2 3 4	4[2 2] 4[1 1]	16	759
1 2 3 5	8[2 1]	16	3036
1 2 3 6	2[3 1] 2[2 2] 2[2 0] 2[1 1]	14	3036
1 2 3 11	2[3 0] 6[2 1]	14	2024
1 2 3 4 5	2[2 1] 6[1 1] 6[1 0]	22	2277
1 2 3 4 6	2[2 1] 2[2 0] 4[1 1] 6[1 0]	20	10626
1 2 3 4 11	2[2 0] 6[1 1] 8[1 0]	22	6072
1 2 3 4 14	8[1 1] 8[1 0]	24	1012
1 2 3 4 19	2[2 1] 4[2 0] 2[1 1] 6[1 0]	18	5060
1 2 3 5 14	4[2 0] 4[1 1] 8[1 0]	20	6831
1 2 3 6 7	2[3 0] 6[1 1] 6[1 0]	20	759
1 2 4 5 10	1[3 0] 1[2 1] 2[2 0] 4[1 1] 6[1 0]	19	506
1 2 5 7 14	6[2 0] 2[1 1] 8[1 0]	18	506

Table 13: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.33
main	100.00	
main and 2fi	60.87	39.13
2fi	64.29	35.71
main and 3fi	62.73	37.27
2fi and 3fi	62.73	37.27
3fi	62.95	37.05

Table 14: Unique runs when projected into 4 dimensions.

n	Columns
16	1:4 14
14	1:23

3.6 The 28 Run Plackett-Burman Design

Table 15: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[7 7]	4	351
1 2 3	4[4 3]	8	2574
1 2 11	4[5 2]	8	351
1 2 3 4	4[3 2] 1[2 0] 3[1 1]	15	1404
1 2 3 7	1[3 1] 3[2 2] 4[2 1]	16	8775
1 2 3 10	3[3 1] 1[3 0] 1[2 2] 3[2 1]	15	5616
1 2 4 18	1[4 0] 1[3 1] 2[3 0] 2[2 2] 2[2 1]	13	1053
1 2 4 23	1[4 1] 3[3 2] 3[2 0] 1[1 1]	13	702

Table 16: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$				
Between	0.00	0.14	0.29	0.43	0.57
main	100.00				
main and 2fi	7.41	81.48		11.11	
2fi	14.29	75.43		10.29	
main and 3fi	11.11	78.22		10.67	
2fi and 3fi	48.03	18.05	30.77	2.46	0.68
3fi	43.99	24.92	27.09	3.40	0.60

Table 17: Unique runs when projected into 4 dimensions.

n	Columns
16	1:3 7 8
15	1:9
13	1:27

3.7 The 32 Run Plackett-Burman Design

Table 18: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[8 8]	4	465
1 2 3	4[4 4]	8	4340
1 2 19	4[8 0]	4	155
1 2 3 4	8[2 2]	16	26040
1 2 3 6	8[4 0]	8	4340
1 2 3 12	4[4 4]	8	1085
1 2 3 4 5	16[1 1]	32	83328
1 2 3 4 6	16[2 0]	16	57288
1 2 3 4 9	8[2 2]	16	26040
1 2 3 6 12	8[4 0]	8	3255

Table 19: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$	
Between	0.00	1.00
main	100.00	
main and 2fi	96.77	3.23
2fi	96.98	3.02
main and 3fi	96.89	3.11
2fi and 3fi	96.89	3.11
3fi	96.90	3.10

Table 20: Unique runs when projected into 3 dimensions.

n	Columns
8	1:5 9 10 12:14 16 18 23 16 28 29
4	1:31

Table 21: Unique runs when projected into 4 dimensions.

n	Columns
16	1:5 15
8	1:31

3.8 The 36 Run Plackett-Burman Design

Table 22: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[9 9]	4	595
1 2 3	4[5 4]	8	5355
1 2 8	4[6 3]	8	1190
1 2 3 4	1[4 1] 3[3 2] 3[3 1] 1[2 2]	16	22680
1 2 3 5	4[3 2] 1[3 1] 3[2 2]	16	15645
1 2 3 9	3[4 2] 1[3 3] 1[3 0] 3[2 1]	15	1890
1 2 3 15	1[4 2] 3[3 3] 4[2 1]	16	5985
1 2 3 18	1[5 1] 1[4 2] 2[3 3] 2[3 0] 2[2 1]	14	1260
1 2 3 25	2[4 1] 1[4 0] 2[3 2] 1[3 1] 2[2 2]	15	4095
1 2 4 17	1[5 0] 1[4 0] 3[3 2] 3[3 1]	14	420
1 2 8 12	1[6 0] 3[3 3] 4[3 0]	11	385

Table 23: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.11	0.22	0.33	0.44	0.67
main	100.00					
main and 2fi	5.71	77.14		17.14		
2fi	11.11	72.73		16.16		
main and 3fi	8.57	74.81		16.62		
2fi and 3fi	44.62	13.20	36.97	2.93	2.26	0.02
3fi	41.57	18.60	33.61	4.13	2.06	0.02

Table 24: Unique runs when projected into 4 dimensions.

n	Columns
16	1:3 5 14 17 21 23 29
15	1:5 9:11 16 22 32 33
14	1:11 22 23 32
11	1:35

3.9 The 40 Run Plackett-Burman Design

Table 25: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[10 10]	4	703
1 2 3	4[6 4]	8	3648
1 2 13	4[8 2]	8	228
1 2 20	4[5 5]	8	4560
1 2 3 4	4[4 2] 1[4 0] 3[2 2]	15	2736
1 2 3 7	1[6 2] 3[4 4] 4[2 0]	12	228
1 2 3 13	1[6 0] 3[4 2] 3[4 0] 1[2 2]	12	912
1 2 3 20	8[3 2]	16	25536
1 2 13 20	8[4 1]	16	1596
1 2 20 21	4[5 5]	8	171
1 2 22 23	2[4 2] 2[3 3] 2[3 1] 2[2 2]	16	19152
1 2 22 26	2[5 3] 2[4 4] 2[2 0] 2[1 1]	14	1368
1 2 22 33	2[5 1] 2[4 2] 2[4 0] 2[3 1]	14	2736
1 21 22 23	2[4 1] 6[3 2]	16	12768
1 21 22 32	2[5 0] 4[4 1] 2[3 2]	14	2736
20 21 22 23	4[3 3] 4[2 2]	16	3648
20 21 22 26	4[4 4] 4[1 1]	16	228

Table 26: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.20	0.40	0.60	1.00
main	100.00				
main and 2fi	56.47	40.97		2.56	
2fi	62.09	35.48		2.22	0.21
main and 3fi	61.10	36.42		2.28	0.21
2fi and 3fi	73.45	16.30		9.23	1.02
3fi	70.81	20.67	7.18	1.29	0.05

3.10 The 44 Run Plackett-Burman Design

Table 27: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[11 11]	4	903
1 2 3	4[6 5]	8	9331
1 2 8	4[7 4]	8	3010
1 2 3 4	1[4 2] 3[3 3] 4[3 2]	16	31605
1 2 3 8	1[5 1] 1[4 2] 2[4 1] 2[3 3] 2[3 2]	16	24381
1 2 3 9	1[5 2] 3[4 3] 3[3 1] 1[2 2]	16	16254
1 2 3 11	2[5 2] 2[4 3] 1[4 0] 1[3 1] 2[2 2]	15	5418
1 2 3 13	3[4 2] 1[4 1] 1[3 3] 3[3 2]	16	33712
1 2 3 17	4[4 3] 1[3 1] 3[2 2]	16	8428
1 2 3 27	1[5 1] 1[5 0] 3[4 2] 3[3 2]	15	3612

Table 28: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$				
Between	0.00	0.09	0.18	0.27	0.36
main	100.00				
main and 2fi	4.65	72.09		23.26	
2fi	9.09	68.74		22.17	
main and 3fi	6.98	70.33		22.69	
2fi and 3fi	42.95	10.05	38.73	3.24	5.02
3fi	40.48	14.34	35.90	4.63	4.66

Table 29: Unique runs when projected into 4 dimensions.

n	Columns
16	1:4 7 9 16 25 30 39
15	1:43

3.11 The 48 Run Plackett-Burman Design

Table 30: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[12 12]	4	1081
1 2 3	4[6 6]	8	6486
1 2 4	4[7 5]	8	8648
1 2 11	4[8 4]	8	1081
1 2 3 4	4[4 2] 4[3 3]	16	42159
1 2 3 5	4[4 3] 4[3 2]	16	28106
1 2 3 6	2[5 2] 2[4 3] 2[4 1] 2[3 2]	16	12972
1 2 3 7	1[5 2] 3[4 3] 1[4 1] 3[3 2]	16	43240
1 2 3 12	2[5 1] 4[4 2] 2[3 3]	16	12972
1 2 3 36	2[5 3] 2[4 4] 2[3 1] 2[2 2]	16	6486
1 2 4 12	1[5 1] 4[4 2] 3[3 3]	16	10810
1 2 4 23	4[5 3] 1[4 0] 3[2 2]	15	1081
1 2 4 27	1[6 2] 2[5 3] 1[4 4] 1[4 0] 2[3 1] 1[2 2]	15	2162
1 2 4 31	3[5 2] 1[5 0] 1[4 3] 3[3 2]	15	4324
1 2 4 32	1[6 1] 1[5 2] 1[5 0] 2[4 3] 1[4 1] 2[3 2]	15	2162
1 2 5 14	8[4 2]	16	4324
1 2 5 37	4[4 4] 4[2 2]	16	1081
1 2 7 11	1[6 1] 3[4 3] 3[4 1] 1[3 2]	16	4324
1 2 7 38	1[6 2] 3[4 4] 4[3 1]	16	1081
1 2 11 33	1[6 0] 2[5 1] 3[4 2] 2[3 3]	15	1081

Table 31: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.17	0.33	0.50
main	100.00			
main and 2fi	42.55	51.06	6.38	
2fi	45.00	48.89	6.11	
main and 3fi	43.83	49.93	6.24	
2fi and 3fi	46.30	46.85	6.73	0.12
3fi	46.21	46.98	6.69	0.12

Table 32: Unique runs when projected into 4 dimensions.

n	Columns
16	1:5 9:10 15 38:40
15	1:47

3.12 The 52 Run Plackett-Burman Design

Table 33: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[13 13]	4	1275
1 2 3	4[12 1]	8	25
1 2 4	4[7 6]	8	16400
2 4 6	4[8 5]	8	3600
2 4 16	4[9 4]	8	800
1 2 3 4	1[7 0] 3[6 1] 3[6 0] 1[5 1]	12	1200
1 2 4 6	1[5 2] 3[4 3] 3[4 2] 1[3 3]	16	69000
1 2 4 7	1[5 3] 3[4 4] 4[3 2]	16	17250
1 2 4 12	4[4 3] 1[4 2] 3[3 3]	16	72000
1 2 4 16	3[5 2] 1[5 1] 1[4 3] 3[4 2]	16	18400
1 2 4 17	4[5 4] 1[3 1] 3[2 2]	16	4600
2 3 4 5	1[7 5] 3[6 6] 4[1 0]	12	300
2 4 6 8	1[7 1] 3[4 4] 4[4 1]	16	150
2 4 6 12	2[6 3] 2[5 4] 1[4 0] 1[3 1] 2[2 2]	15	1800
2 4 6 14	1[6 2] 1[5 3] 2[4 4] 2[4 1] 2[3 2]	16	4800
2 4 6 16	1[6 1] 1[6 0] 2[5 2] 1[4 3] 2[4 2] 1[3 3]	15	3600
2 4 6 24	1[6 1] 1[5 1] 3[4 3] 3[4 2]	16	3200
2 4 6 30	2[6 2] 1[5 3] 1[5 0] 1[4 4] 1[4 1] 2[3 2]	15	3600
2 4 7 13	1[6 3] 3[5 4] 3[3 1] 1[2 2]	16	2400
2 4 7 21	3[5 3] 1[4 4] 1[4 1] 3[3 2]	16	14400
2 4 7 27	2[5 2] 1[5 1] 2[4 3] 1[4 2] 2[3 3]	16	21600
2 4 7 31	1[6 2] 3[5 3] 3[4 1] 1[3 2]	16	2400
2 4 9 23	1[6 1] 1[5 2] 2[5 1] 2[4 3] 1[4 2] 1[3 3]	16	7200
2 4 14 16	1[8 0] 1[5 3] 2[5 0] 2[4 4] 2[4 1]	13	300
2 4 14 48	1[8 1] 3[5 4] 3[4 0] 1[3 1]	13	200
2 4 15 49	1[7 2] 1[6 3] 2[5 4] 2[4 0] 1[3 1] 1[2 2]	14	600
2 4 25 33	1[7 1] 1[6 2] 2[5 0] 2[4 4] 2[3 2]	14	300
2 5 15 17	1[7 0] 1[6 0] 1[5 2] 1[5 1] 2[4 3] 2[4 2]	14	600

Table 34: Correlation structure (% cases).

				$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.08	0.15	0.23	0.31	0.38	0.46	0.77	0.85
main	100.00								
main and 2fi	3.92	75.66		16.61		3.69			0.12
2fi	7.69	72.69		15.96		3.55			0.11
main and 3fi	5.88	74.12		16.27		3.62			0.11
2fi and 3fi	44.30	8.89	36.89	1.95	6.69	0.43	0.81	0.02	0.01

3.13 The 56 Run Plackett-Burman Design

Table 35: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[14 14]	4	1431
1 2 3	4[8 6]	8	10296
1 2 11	4[10 4]	8	1404
1 2 28	4[7 7]	8	13104
1 2 3 4	4[6 4] 1[4 0] 3[2 2]	15	1404
1 2 3 7	1[6 2] 3[4 4] 4[4 2]	16	8775
1 2 3 10	3[6 2] 1[6 0] 1[4 4] 3[4 2]	15	5616
1 2 3 28	8[4 3]	16	92664
1 2 4 18	1[8 0] 1[6 2] 2[6 0] 2[4 4] 2[4 2]	13	1053
1 2 4 23	1[8 2] 3[6 4] 3[4 0] 1[2 2]	13	702
1 2 11 28	8[5 2]	16	12636
1 2 28 29	4[7 7]	8	351
1 2 30 31	2[6 4] 2[5 5] 2[3 1] 2[2 2]	16	10530
1 2 30 34	2[5 3] 2[4 4] 2[4 2] 2[3 3]	16	70551
1 2 30 37	2[6 2] 2[5 3] 2[5 1] 2[4 2]	16	21060
1 2 31 50	2[7 3] 2[6 4] 2[4 0] 2[3 1]	14	2106
1 2 37 52	2[7 1] 2[6 0] 2[4 4] 2[3 3]	14	1053
1 29 30 31	2[5 2] 6[4 3]	16	47034
1 29 30 37	2[6 1] 4[5 2] 2[4 3]	16	21060
1 29 34 38	2[7 0] 2[6 1] 4[4 3]	14	2106
28 29 30 31	4[5 5] 4[2 2]	16	2106
28 29 30 34	4[4 4] 4[3 3]	16	15444

Table 36: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.14	0.29	0.43	0.57	1.00
main	100.00					
main and 2fi	54.58	39.97		5.45		
2fi	58.73	36.23		4.94		0.10
main and 3fi	57.97	36.90		5.03		0.10
2fi and 3fi	72.49	11.40	14.24	1.55	0.32	

3.14 The 60 Run Plackett-Burman Design

Table 37: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[15 15]	4	1711
1 2 3	4[8 7]	8	20532
1 2 7	4[9 6]	8	11977
1 2 3 4	1[5 3] 3[4 4] 4[4 3]	16	42775
1 2 3 6	3[5 3] 1[5 2] 1[4 4] 3[4 3]	16	102660
1 2 3 7	1[6 3] 3[5 4] 3[4 2] 1[3 3]	16	68440
1 2 3 8	1[7 2] 3[5 4] 1[5 1] 3[4 2]	16	20532
1 2 3 11	4[5 4] 1[4 2] 3[3 3]	16	25665
1 2 3 12	2[6 3] 2[5 4] 1[5 1] 1[4 2] 2[3 3]	16	51330
1 2 3 21	1[6 2] 1[5 3] 2[5 2] 2[4 4] 2[4 3]	16	102660
1 2 3 28	1[6 2] 1[6 1] 3[5 3] 3[4 3]	16	34220
1 2 7 28	1[7 1] 4[5 2] 3[4 4]	16	5133
1 2 26 27	4[6 3] 1[6 0] 3[3 3]	15	1711

Table 38: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.07	0.13	0.20	0.27	0.40
main	100.00					
main and 2fi	3.39	61.02		35.59		
2fi	6.67	58.95		34.39		
main and 3fi	5.08	59.95		34.97		
2fi and 3fi	39.43	6.20	40.60	3.62	9.60	0.55

Table 39: Unique runs when projected into 4 dimensions.

n	Columns
16	1:9 12 15 16 22:24 33 36 38 40 49
15	1:59

3.15 The 64 Run Plackett-Burman Design

Table 40: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[16 16]	4	1953
1 2 3	4[8 8]	8	39060
1 2 19	4[16 0]	4	651
1 2 3 4	8[4 4]	16	546840
1 2 3 6	8[8 0]	8	39060
1 2 3 12	4[8 8]	8	9765

Table 41: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	1.00
main	100.00	
main and 2fi	98.41	1.59
2fi	98.46	1.54
main and 3fi	98.44	1.56
2fi and 3fi	98.44	1.56

Table 42: Unique runs when projected into 3 dimensions.

n	Columns
8	1:62
4	1:63

Table 43: Unique runs when projected into 4 dimensions.

n	Columns
16	1:4 6 32 36 38:62
8	1:63

3.16 The 68 Run Plackett-Burman Design

Table 44: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[17 17]	4	2211
1 2 3	4[9 8]	8	28743
1 2 6	4[10 7]	8	18425
1 2 39	4[11 6]	8	737
1 2 3 4	4[5 4] 1[5 3] 3[4 4]	16	55275
1 2 3 6	2[6 3] 1[6 2] 2[5 4] 1[5 3] 2[4 4]	16	132660
1 2 3 7	1[7 3] 1[6 4] 2[5 5] 2[5 2] 2[4 3]	16	112761
1 2 3 8	3[6 4] 1[5 5] 1[5 2] 3[4 3]	16	88440
1 2 3 9	1[6 4] 3[5 5] 4[4 3]	16	39798
1 2 3 10	1[6 3] 3[5 4] 3[5 3] 1[4 4]	16	159192
1 2 3 39	3[6 3] 1[6 2] 1[5 4] 3[5 3]	16	8844
1 2 3 42	1[7 2] 1[6 2] 3[5 4] 3[5 3]	16	75174
1 2 3 44	4[6 5] 1[4 2] 3[3 3]	16	2211
1 2 4 24	2[7 3] 1[6 4] 1[6 1] 1[5 5] 1[5 2] 2[4 3]	16	8844
1 2 4 34	2[7 4] 2[6 5] 1[5 1] 1[4 2] 2[3 3]	16	4422
1 2 5 25	1[7 2] 1[7 1] 2[6 3] 1[5 4] 2[5 3] 1[4 4]	16	8844
1 2 5 29	1[7 4] 3[6 5] 3[4 2] 1[3 3]	16	2948
1 2 5 31	1[7 3] 3[6 4] 1[6 1] 3[4 3]	16	35376
1 2 5 39	1[7 3] 3[6 4] 3[5 2] 1[4 3]	16	2948
1 2 6 17	1[8 1] 3[6 2] 3[5 4] 1[5 3]	16	2211
1 2 6 20	1[8 2] 2[6 4] 1[6 1] 1[5 5] 2[5 2] 1[4 3]	16	6633
1 2 6 31	1[7 1] 4[6 3] 3[4 4]	16	8844
1 2 6 33	1[8 3] 3[6 5] 1[5 1] 3[4 2]	16	2211
1 2 7 39	1[7 2] 1[6 3] 2[6 2] 2[5 4] 1[5 3] 1[4 4]	16	8844

Table 45: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$						
Between	0.00	0.06	0.12	0.18	0.24	0.29	0.35
main	100.00						
main and 2fi	2.99	58.21		37.31		1.49	
2fi	5.88	56.47		36.20		1.45	
main and 3fi	4.48	57.31		36.74		1.47	
2fi and 3fi	37.00	5.21	42.13	3.34	11.06	0.13	1.13

3.17 The 72 Run Plackett-Burman Design

Table 46: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[18 18]	4	2485
1 2 3	4[9 9]	8	19880
1 2 4	4[11 7]	8	9940
1 2 5	4[10 8]	8	27335
1 2 3 4	2[7 3] 2[6 2] 2[5 5] 2[4 4]	16	22365
1 2 3 5	2[7 4] 2[6 5] 2[5 2] 2[4 3]	16	44730
1 2 3 6	8[5 4]	16	39760
1 2 3 7	4[6 3] 4[5 4]	16	89460
1 2 3 8	2[7 2] 2[6 3] 4[5 4]	16	44730
1 2 3 9	1[7 3] 2[6 4] 1[6 2] 1[5 5] 2[5 3] 1[4 4]	16	134190
1 2 3 10	2[6 3] 6[5 4]	16	114310
1 2 3 11	2[6 4] 2[5 5] 2[5 3] 2[4 4]	16	171465
1 2 3 16	4[6 5] 4[4 3]	16	29820
1 2 3 28	4[5 5] 4[4 4]	16	9940
1 2 3 30	4[6 4] 4[5 3]	16	29820
1 2 3 42	1[7 4] 3[6 5] 1[5 2] 3[4 3]	16	44730
1 2 4 5	1[8 2] 1[6 4] 2[6 2] 2[5 5] 2[5 3]	16	29820
1 2 4 8	1[8 2] 1[7 1] 3[6 4] 3[5 3]	16	4970
1 2 4 9	1[8 3] 1[7 4] 2[6 5] 1[6 1] 1[5 2] 2[4 3]	16	14910
1 2 4 13	1[8 3] 3[6 5] 3[5 2] 1[4 3]	16	19880
1 2 4 19	1[7 2] 3[6 3] 4[5 4]	16	44730
1 2 4 22	1[8 1] 1[7 2] 3[6 3] 3[5 4]	16	4970
1 2 4 41	2[7 3] 1[7 1] 2[6 4] 1[5 3] 2[4 4]	16	22365
1 2 4 42	3[7 4] 1[6 5] 1[6 1] 3[4 3]	16	14910
1 2 5 41	4[6 4] 1[6 2] 3[4 4]	16	39760

Table 47: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$			
Between	0.00	0.11	0.22	0.33
main	100.00			
main and 2fi	36.62	46.48	16.90	
2fi	38.41	45.17	16.43	
main and 3fi	37.54	45.81	16.66	
2fi and 3fi	39.02	46.04	13.61	1.33

3.18 The 76 Run Plackett-Burman Design

Table 48: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[19 19]	4	2775
1 2 3	4[18 1]	8	37
1 2 4	4[10 9]	8	49728
2 4 10	4[11 8]	8	10656
2 4 14	4[12 7]	8	7104
1 2 3 4	1[10 0] 3[9 1] 3[9 0] 1[8 1]	12	2664
1 2 4 6	1[6 4] 3[5 5] 4[5 4]	16	264735
1 2 4 10	3[6 4] 1[6 3] 1[5 5] 3[5 4]	16	231768
1 2 4 11	4[6 5] 1[5 3] 3[4 4]	16	57942
1 2 4 14	1[7 3] 3[6 4] 3[6 3] 1[5 4]	16	144744
1 2 4 15	1[7 5] 3[6 6] 4[4 3]	16	36186
2 3 4 5	1[10 8] 3[9 9] 4[1 0]	12	666
2 4 6 12	2[7 4] 2[6 5] 1[6 2] 1[5 3] 2[4 4]	16	20646
2 4 6 14	2[7 3] 1[7 2] 1[6 4] 1[6 3] 1[5 5] 2[5 4]	16	103896
2 4 6 16	1[10 2] 3[7 5] 1[7 0] 3[5 2]	15	1332
2 4 6 18	2[8 4] 1[7 5] 1[6 6] 1[6 1] 1[5 2] 2[4 3]	16	17316
2 4 6 22	1[7 3] 1[7 2] 3[6 4] 3[5 4]	16	13764
2 4 6 26	1[8 2] 1[8 1] 1[7 3] 2[6 4] 1[6 3] 2[5 4]	16	17316
2 4 6 30	3[7 4] 1[6 5] 1[6 2] 3[5 3]	16	34632
2 4 6 36	2[8 3] 1[7 1] 2[6 5] 1[6 2] 2[4 4]	16	8658
2 4 6 38	3[7 5] 1[6 6] 1[5 2] 3[4 3]	16	34632
2 4 7 13	1[7 4] 3[6 5] 3[5 3] 1[4 4]	16	38628
2 4 7 17	1[9 3] 1[8 4] 2[6 6] 2[6 1] 2[4 3]	16	3996
2 4 7 23	1[7 3] 1[6 4] 2[6 3] 2[5 5] 2[5 4]	16	57942
2 4 7 27	1[8 2] 1[7 3] 2[7 2] 2[5 5] 2[5 4]	16	27972
2 4 7 59	1[8 4] 3[7 5] 3[5 2] 1[4 3]	16	18648
2 4 8 26	1[8 4] 1[7 5] 2[6 6] 2[5 2] 2[4 3]	16	13320
2 4 8 32	1[9 3] 2[7 5] 1[6 6] 1[6 1] 2[5 2] 1[4 3]	16	1332
2 4 8 34	1[8 3] 1[7 4] 2[6 5] 2[6 2] 1[5 3] 1[4 4]	16	26640
2 4 8 46	1[9 2] 1[7 4] 1[7 1] 2[6 5] 1[6 2] 2[5 3]	16	1332
2 4 8 58	1[8 2] 1[7 2] 2[6 4] 2[6 3] 1[5 5] 1[5 4]	16	26640
2 4 10 12	1[9 3] 3[6 6] 4[5 2]	16	666
2 4 10 24	3[8 4] 1[7 5] 1[7 0] 3[4 3]	15	444
2 4 10 34	1[9 2] 3[6 5] 3[6 2] 1[5 3]	16	2664
2 4 10 70	2[8 3] 1[8 0] 2[7 4] 1[5 3] 2[4 4]	15	666
2 4 12 14	1[9 1] 2[7 2] 1[6 4] 2[6 3] 2[5 5]	16	666
2 4 14 16	1[10 0] 4[7 2] 3[5 5]	15	333
2 5 15 17	1[9 1] 1[8 1] 3[6 4] 3[6 3]	16	2664

Table 49: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.05	0.11	0.16	0.21	0.26	0.32	0.53	0.89
main	100.00								
main and 2fi	2.67	71.68		15.36		10.24			0.05
2fi	5.26	69.77		14.95		9.97			0.05
main and 3fi	4.00	70.70		15.15		10.10			0.05
2fi and 3fi	43.72	5.73	36.10	1.23	7.90	0.82	4.44	0.06	0.00

3.19 The 80 Run Plackett-Burman Design

Table 50: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[20 20]	4	3081
1 2 3	4[10 10]	8	22594
1 2 5	4[11 9]	8	41080
1 2 9	4[12 8]	8	15405
1 2 3 4	4[6 5] 4[5 4]	16	67782
1 2 3 6	2[7 5] 2[6 6] 2[5 3] 2[4 4]	16	77025
1 2 3 8	4[6 4] 4[5 5]	16	101673
1 2 3 9	2[7 4] 2[6 5] 2[6 3] 2[5 4]	16	154050
1 2 3 10	1[8 4] 2[7 5] 1[6 6] 1[6 2] 2[5 3] 1[4 4]	16	129402
1 2 3 12	1[8 3] 1[7 4] 1[7 2] 2[6 5] 1[6 3] 2[5 4]	16	129402
1 2 3 13	1[7 4] 3[6 5] 1[6 3] 3[5 4]	16	262912
1 2 3 19	2[7 3] 4[6 4] 2[5 5]	16	154050
1 2 3 35	8[6 4]	16	36972
1 2 3 69	4[6 6] 4[4 4]	16	9243
1 2 5 7	4[7 5] 1[6 2] 3[4 4]	16	18486
1 2 5 10	1[8 3] 3[6 5] 3[6 3] 1[5 4]	16	73944
1 2 5 11	3[7 4] 1[7 2] 1[6 5] 3[5 4]	16	73944
1 2 5 12	1[8 4] 3[6 6] 4[5 3]	16	18486
1 2 5 15	1[7 3] 4[6 4] 3[5 5]	16	65728
1 2 5 33	1[8 2] 2[7 3] 3[6 4] 2[5 5]	16	64701
1 2 8 9	2[8 4] 2[6 6] 2[6 2] 2[4 4]	16	9243
1 2 8 71	2[8 2] 6[6 4]	16	6162
1 2 9 18	1[9 2] 1[7 2] 3[6 5] 3[6 3]	16	6162
1 2 9 24	1[9 3] 1[7 5] 2[6 6] 2[6 2] 2[5 3]	16	9243
1 2 9 29	2[8 4] 2[7 5] 1[7 1] 1[5 3] 2[4 4]	16	18486
1 2 9 75	1[8 3] 1[8 1] 3[7 4] 3[5 4]	16	12324
1 2 30 31	1[10 2] 3[6 6] 4[6 2]	16	3081

Table 51: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.10	0.20	0.30	0.40
main	100.00				
main and 2fi	30.38	50.63	18.99		
2fi	32.14	49.35	18.51		
main and 3fi	31.28	49.98	18.74		
2fi and 3fi	37.43	45.30	15.20	2.05	0.03

3.20 The 84 Run Plackett-Burman Design

Table 52: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[21 21]	4	3403
1 2 3	4[11 10]	8	47642
1 2 4	4[12 9]	8	40836
1 2 23	4[13 8]	8	3403
1 2 3 4	2[7 4] 1[7 3] 2[6 5] 1[6 4] 2[5 5]	16	357315
1 2 3 5	3[7 5] 1[6 6] 1[6 3] 3[5 4]	16	238210
1 2 3 9	1[7 4] 3[6 5] 3[6 4] 1[5 5]	16	176956
1 2 3 11	2[8 4] 1[7 5] 1[7 2] 1[6 6] 1[6 3] 2[5 4]	16	34030
1 2 3 12	1[8 4] 1[7 5] 2[6 6] 2[6 3] 2[5 4]	16	255225
1 2 3 17	1[8 3] 1[7 3] 3[6 5] 3[6 4]	16	170150
1 2 3 19	4[6 5] 1[6 4] 3[5 5]	16	85075
1 2 3 22	1[7 5] 3[6 6] 4[5 4]	16	44239
1 2 3 23	1[8 3] 1[7 4] 2[7 3] 2[6 5] 1[6 4] 1[5 5]	16	71463
1 2 3 28	1[8 5] 3[7 6] 3[5 3] 1[4 4]	16	23821
1 2 3 48	2[8 5] 2[7 6] 1[6 2] 1[5 3] 2[4 4]	16	17015
1 2 3 51	1[8 3] 1[8 2] 2[7 4] 1[6 5] 2[6 4] 1[5 5]	16	34030
1 2 3 52	1[8 4] 3[7 5] 3[6 3] 1[5 4]	16	23821
1 2 4 8	1[8 4] 3[7 5] 1[7 2] 3[5 4]	16	122508
1 2 4 12	1[9 2] 3[7 3] 3[6 5] 1[6 4]	16	17015
1 2 4 16	1[8 2] 4[7 4] 3[5 5]	16	30627
1 2 4 51	1[9 3] 2[7 5] 1[7 2] 1[6 6] 2[6 3] 1[5 4]	16	51045
1 2 4 63	1[9 3] 3[6 6] 4[6 3]	16	34030
1 2 4 73	1[9 4] 3[7 6] 1[6 2] 3[5 3]	16	17015
1 2 5 51	3[8 5] 1[7 6] 1[6 2] 3[5 3]	16	6806
1 2 5 68	2[8 3] 1[8 2] 1[7 3] 2[6 5] 2[5 5]	16	10209
1 2 6 13	4[8 5] 1[7 1] 3[4 4]	16	3403
1 2 6 36	3[8 4] 1[8 1] 1[7 5] 3[5 4]	16	13612

Table 53: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$							
Between	0.00	0.05	0.10	0.14	0.19	0.24	0.29	0.38
main	100.00							
main and 2fi	2.41	50.60		43.37		3.61		
2fi	4.76	49.38		42.33		3.53		
main and 3fi	3.61	49.98		42.84		3.57		
2fi and 3fi	33.87	3.66	42.71	3.13	13.78	0.26	2.53	0.07

3.21 The 88 Run Plackett-Burman Design

Table 54: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[22 22]	4	3655
1 2 3	4[12 10]	8	37324
1 2 8	4[14 8]	8	12040
1 2 44	4[11 11]	8	52976
1 2 3 4	1[8 4] 3[6 6] 4[6 4]	16	31605
1 2 3 8	1[10 2] 1[8 4] 2[8 2] 2[6 6] 2[6 4]	16	24381
1 2 3 9	1[10 4] 3[8 6] 3[6 2] 1[4 4]	16	16254
1 2 3 11	2[10 4] 2[8 6] 1[8 0] 1[6 2] 2[4 4]	15	5418
1 2 3 13	3[8 4] 1[8 2] 1[6 6] 3[6 4]	16	33712
1 2 3 17	4[8 6] 1[6 2] 3[4 4]	16	8428
1 2 3 27	1[10 2] 1[10 0] 3[8 4] 3[6 4]	15	3612
1 2 3 44	8[6 5]	16	485212
1 2 8 44	8[7 4]	16	156520
1 2 44 45	4[11 11]	8	903
1 2 46 47	2[7 5] 2[6 6] 2[6 4] 2[5 5]	16	315147
1 2 46 51	2[8 4] 2[7 5] 2[7 3] 2[6 4]	16	209496
1 2 46 52	2[9 5] 2[8 6] 2[6 2] 2[5 3]	16	70434
1 2 46 60	2[8 6] 2[7 7] 2[5 3] 2[4 4]	16	104748
1 2 46 64	2[10 4] 2[7 7] 2[7 1] 2[4 4]	16	5418
1 2 46 70	2[9 3] 2[8 2] 2[6 6] 2[5 5]	16	35217
1 45 46 47	2[7 4] 6[6 5]	16	210098
1 45 46 51	2[9 2] 2[8 3] 4[6 5]	16	70434
1 45 46 56	2[8 3] 4[7 4] 2[6 5]	16	209496
1 45 54 78	2[10 1] 6[7 4]	16	3612
44 45 46 47	4[6 6] 4[5 5]	16	93310
44 45 46 52	4[7 7] 4[4 4]	16	30100

Table 55: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.09	0.18	0.27	0.36	1.00
main	100.00					
main and 2fi	52.89	35.62		11.49		
2fi	55.61	33.54		10.82		0.04
main and 3fi	55.09	33.93		10.94		0.04
2fi and 3fi	70.58	6.48	18.46	2.09	2.39	

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Table 56: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[24 24]	4	4371
1 2 3	4[12 12]	8	95128
1 2 4	4[14 10]	8	34592
1 2 11	4[16 8]	8	4324
1 2 3 4	4[8 4] 4[6 6]	16	405375
1 2 3 5	4[8 6] 4[6 4]	16	270250
1 2 3 6	2[10 4] 2[8 6] 2[8 2] 2[6 4]	16	56212
1 2 3 7	1[10 4] 3[8 6] 1[8 2] 3[6 4]	16	43240
1 2 3 12	2[10 2] 4[8 4] 2[6 6]	16	56212
1 2 3 36	2[10 6] 2[8 8] 2[6 2] 2[4 4]	16	28106
1 2 3 48	8[6 6]	16	526447
1 2 4 12	1[10 2] 4[8 4] 3[6 6]	16	10810
1 2 4 23	4[10 6] 1[8 0] 3[4 4]	15	1081
1 2 4 27	1[12 4] 2[10 6] 1[8 8] 1[8 0] 2[6 2] 1[4 4]	15	2162
1 2 4 31	3[10 4] 1[10 0] 1[8 6] 3[6 4]	15	4324
1 2 4 32	1[12 2] 1[10 4] 1[10 0] 2[8 6] 1[8 2] 2[6 4]	15	2162
1 2 4 48	8[7 5]	16	873448
1 2 5 14	8[8 4]	16	116748
1 2 5 37	4[8 8] 4[4 4]	16	25944
1 2 7 11	1[12 2] 3[8 6] 3[8 2] 1[6 4]	16	4324
1 2 7 38	1[12 4] 3[8 8] 4[6 2]	16	1081
1 2 11 33	1[12 0] 2[10 2] 3[8 4] 2[6 6]	15	1081
1 2 48 49	4[12 12]	8	1081
1 2 50 53	4[9 5] 4[7 3]	16	30268
1 2 50 54	2[9 5] 2[7 7] 2[7 3] 2[5 5]	16	155664
1 2 50 56	4[7 7] 4[5 5]	16	192418
1 2 50 59	4[9 3] 4[7 5]	16	90804
1 2 50 83	4[9 7] 4[5 3]	16	30268
1 2 51 79	2[11 3] 2[9 1] 2[7 7] 2[5 5]	16	2162
1 2 52 80	2[11 5] 2[9 7] 2[7 1] 2[5 3]	16	4324
1 2 54 73	4[10 6] 4[6 2]	16	2162
1 2 76 93	4[10 2] 4[6 6]	16	3243
1 49 51 53	2[9 3] 6[7 5]	16	103776
1 49 54 63	2[11 1] 2[9 3] 4[7 5]	16	4324

Table 57: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.17	0.33	0.50	1.00
main	100.00				
main and 2fi	71.59	25.26	3.16		
2fi	73.07	23.90	2.99		0.03
main and 3fi	72.79	24.16	3.02		0.03
2fi and 3fi	73.07	23.52	3.36	0.06	

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Table 58: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[25 25]	4	4851
1 2 3	4[24 1]	8	49
1 2 4	4[13 12]	8	114464
2 4 6	4[16 9]	8	4704
2 4 18	4[15 10]	8	18816
2 4 30	4[14 11]	8	18816
1 2 3 4	1[13 0] 3[12 1] 3[12 0] 1[11 1]	12	4704
1 2 4 6	1[9 4] 3[8 5] 3[8 4] 1[7 5]	16	131712
1 2 4 7	1[9 7] 3[8 8] 4[5 4]	16	32928
1 2 4 8	4[7 6] 1[7 5] 3[6 6]	16	754600
1 2 4 18	3[8 5] 1[8 4] 1[7 6] 3[7 5]	16	536256
1 2 4 19	4[8 7] 1[6 4] 3[5 5]	16	134064
1 2 4 30	1[8 5] 3[7 6] 3[7 5] 1[6 6]	16	545664
1 2 4 31	1[8 6] 3[7 7] 4[6 5]	16	136416
2 3 4 5	1[13 11] 3[12 12] 4[1 0]	12	1176
2 4 6 8	1[12 4] 1[11 5] 2[8 8] 2[8 1] 2[5 4]	16	2352
2 4 6 10	1[12 1] 1[11 1] 3[8 5] 3[8 4]	16	1568
2 4 6 16	1[11 4] 1[10 5] 2[8 7] 2[8 2] 1[6 4] 1[5 5]	16	14112
2 4 6 18	2[10 4] 1[9 2] 1[8 6] 1[8 3] 1[7 7] 2[6 5]	16	18816
2 4 6 20	1[11 2] 1[10 2] 2[8 5] 2[8 4] 1[7 6] 1[7 5]	16	14112
2 4 6 22	1[12 2] 1[9 2] 2[8 6] 2[8 3] 1[7 7] 1[7 4]	16	4704
2 4 6 30	1[10 4] 1[9 5] 2[8 6] 2[8 3] 1[7 4] 1[6 5]	16	14112
2 4 6 32	1[12 3] 1[9 6] 2[8 7] 2[8 2] 1[7 3] 1[6 4]	16	4704
2 4 6 34	1[10 3] 1[10 2] 1[9 4] 1[8 5] 1[8 4] 1[7 6] 1[7 5] 1[6 6]	16	37632
2 4 6 44	1[10 3] 1[9 3] 2[8 5] 2[8 4] 1[7 6] 1[6 6]	16	14112
2 4 6 52	2[10 5] 1[9 6] 1[8 7] 1[8 2] 1[7 3] 2[6 4]	16	18816
2 4 7 9	3[10 6] 1[9 7] 1[7 2] 3[6 3]	16	4704
2 4 7 13	2[10 3] 1[10 2] 1[9 3] 2[7 6] 2[6 6]	16	7056
2 4 7 17	3[9 6] 1[8 7] 1[7 3] 3[6 4]	16	70560
2 4 7 23	2[9 5] 1[8 6] 1[8 3] 1[7 7] 1[7 4] 2[6 5]	16	42336
2 4 7 29	1[9 5] 3[8 6] 3[7 4] 1[6 5]	16	84672
2 4 7 33	1[10 5] 1[9 6] 2[8 7] 2[7 3] 1[6 4] 1[5 5]	16	47040
2 4 7 35	1[9 4] 1[8 5] 2[8 4] 2[7 6] 1[7 5] 1[6 6]	16	254016
2 4 7 41	3[8 6] 1[7 7] 1[7 4] 3[6 5]	16	65856
2 4 7 45	2[8 5] 1[8 4] 2[7 6] 1[7 5] 2[6 6]	16	98784
2 4 7 53	1[9 6] 3[8 7] 3[6 4] 1[5 5]	16	84672
2 4 8 18	1[10 4] 1[9 5] 1[9 2] 2[8 6] 1[7 4] 2[6 5]	16	9408
2 4 8 26	1[10 5] 2[9 6] 1[8 7] 1[8 2] 2[6 4] 1[5 5]	16	9408
2 4 9 27	2[9 6] 2[8 7] 1[7 3] 1[6 4] 2[5 5]	16	21168
2 4 9 37	1[10 4] 1[9 5] 2[8 3] 2[7 7] 2[6 5]	16	23520
2 4 11 45	2[9 4] 1[9 3] 1[8 4] 2[7 6] 2[6 6]	16	105840
2 4 16 32	1[10 6] 1[9 7] 2[8 8] 2[6 3] 2[5 4]	16	7056
2 4 16 36	2[10 6] 1[9 7] 1[8 8] 1[7 2] 1[6 3] 2[5 4]	16	18816
2 4 16 38	1[9 5] 1[8 6] 2[7 7] 2[7 4] 2[6 5]	16	28224

Table 59: Repeat and mirror-image patterns (cont).

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
2 4 16 46	1[10 3] 1[9 3] 1[8 5] 1[8 4] 2[7 6] 2[7 5]	16	47040
2 4 16 80	1[9 4] 1[8 4] 3[7 6] 3[7 5]	16	18816
2 4 16 98	1[11 5] 1[10 6] 2[8 8] 2[7 2] 2[5 4]	16	7056
2 4 17 31	2[9 4] 1[9 3] 1[8 5] 1[8 4] 1[7 6] 2[7 5]	16	56448
2 4 17 33	3[9 7] 1[8 8] 1[6 3] 3[5 4]	16	18816
2 4 17 37	1[10 6] 3[9 7] 3[6 3] 1[5 4]	16	18816
2 4 17 59	1[10 3] 1[9 4] 2[9 3] 2[7 6] 1[7 5] 1[6 6]	16	56448
2 4 18 20	1[11 1] 4[9 4] 3[6 6]	16	1176
2 4 18 26	1[10 2] 2[9 4] 2[8 5] 1[7 5] 2[6 6]	16	4704
2 4 18 30	3[9 5] 1[9 2] 1[8 6] 3[6 5]	16	4704
2 4 18 64	1[11 4] 3[9 6] 1[9 1] 3[6 4]	16	4704
2 4 19 35	3[9 5] 1[8 6] 1[8 3] 3[7 4]	16	18816
2 4 19 61	1[10 5] 3[9 6] 3[7 3] 1[6 4]	16	18816
2 4 19 65	2[10 5] 2[8 7] 1[8 2] 1[7 3] 2[5 5]	16	21168
2 4 20 22	4[10 5] 1[10 0] 3[5 5]	15	1176
2 4 20 24	1[12 4] 1[9 7] 2[8 8] 2[7 2] 2[6 3]	16	2352
2 4 30 32	4[9 6] 1[8 2] 3[5 5]	16	1176
2 5 19 21	1[10 3] 1[10 2] 3[8 5] 3[7 5]	16	14112
2 5 19 31	1[9 4] 1[9 3] 2[8 5] 1[7 6] 2[7 5] 1[6 6]	16	42336

Table 60: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$											
Between	0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.40	0.56	0.92
main	100.00											
main and 2fi	2.02	71.50		11.75		11.75		2.94				0.03
2fi	4.00	70.06		11.52		11.52		2.88				0.03
main and 3fi	3.03	70.77		11.63		11.63		2.91				0.03
2fi and 3fi	43.87	4.33	32.72	0.71	10.68	0.71	5.77	0.18	0.74	0.27	0.01	0.00

4 Foldovers of the Plackett-Burman Design

4.1 Foldover of the 12 Run Plackett-Burman Design

Table 61: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[6 6]	4	66
1 2 3	4[3 3]	8	220
1 2 3 4	4[2 2] 4[1 1]	16	495
1 2 3 4 5	1[2 2] 10[1 1]	22	792
1 2 3 4 5 6	12[1 1]	24	792
1 2 3 4 5 10	1[2 2] 10[1 1]	22	132

Table 62: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$		
Between	0.00	0.33	0.67
main	100.00		
main and 2fi	100.00		
2fi	30.77	69.23	
main and 3fi	25.00	75.00	
2fi and 3fi	100.00		
3fi	45.21	49.32	5.48

4.2 Foldover of the 20 Run Plackett-Burman Design

Table 63: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[10 10]	4	190
1 2 3	4[5 5]	8	1140
1 2 3 4	4[3 3] 4[2 2]	16	4560
1 2 3 7	4[4 4] 4[1 1]	16	285
1 2 3 4 5	5[2 2] 10[1 1]	30	10944
1 2 3 4 7	1[3 3] 6[2 2] 5[1 1]	24	4560
1 2 3 4 5 6	1[2 2] 18[1 1]	38	4560
1 2 3 4 5 7	2[2 2] 16[1 1]	36	15390
1 2 3 4 5 9	4[2 2] 12[1 1]	32	5130
1 2 3 4 5 12	3[2 2] 14[1 1]	34	6840
1 2 3 4 5 18	20[1 1]	40	6270
1 2 3 4 7 8	6[2 2] 8[1 1]	28	570

Table 64: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$			
Between	0.00	0.20	0.40	0.60
main	100.00			
main and 2fi	100.00			
2fi	19.05	76.19		4.76
main and 3fi	15.00	80.00		5.00
2fi and 3fi	100.00			
3fi	46.62	33.71	17.56	2.11

4.3 Foldover of the 24 Run Plackett-Burman Design

Table 65: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[12 12]	4	276
1 2 3	4[6 6]	8	2024
1 2 3 4	4[4 4] 4[2 2]	16	4554
1 2 3 5	8[3 3]	16	6072
1 2 3 4 5	2[3 3] 6[2 2] 6[1 1]	28	24288
1 2 3 4 11	8[2 2] 8[1 1]	32	18216
1 2 3 4 5 6	1[3 3] 3[2 2] 15[1 1]	38	2024
1 2 3 4 5 7	4[2 2] 16[1 1]	40	69828
1 2 3 4 5 10	6[2 2] 12[1 1]	36	22264
1 2 3 4 5 14	2[2 2] 20[1 1]	44	33396
1 2 3 4 7 17	2[3 3] 18[1 1]	40	3036
1 2 3 4 11 20	24[1 1]	48	4048

Table 66: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.33
main	100.00	
main and 2fi	100.00	
2fi	64.00	36.00
main and 3fi	62.50	37.50
2fi and 3fi	100.00	
3fi	62.93	37.07

4.4 Foldover of the 28 Run Plackett-Burman Design

Table 67: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[14 14]	4	378
1 2 3	4[7 7]	8	3276
1 2 3 4	4[5 5] 4[2 2]	16	2457
1 2 3 7	4[4 4] 4[3 3]	16	18018
1 2 3 4 5	1[4 4] 3[3 3] 6[2 2] 3[1 1]	26	9828
1 2 3 4 7	4[3 3] 5[2 2] 6[1 1]	30	39312
1 2 3 7 8	1[3 3] 10[2 2] 5[1 1]	32	49140
1 2 3 4 5 6	1[3 3] 9[2 2] 7[1 1]	34	3276
1 2 3 4 5 7	7[2 2] 14[1 1]	42	58968
1 2 3 4 5 10	2[3 3] 4[2 2] 14[1 1]	40	29484
1 2 3 4 5 11	6[2 2] 16[1 1]	44	58968
1 2 3 4 5 12	8[2 2] 12[1 1]	40	44226
1 2 3 4 5 28	1[3 3] 3[2 2] 19[1 1]	46	9828
1 2 3 4 7 8	4[2 2] 20[1 1]	48	73710
1 2 3 4 7 15	5[2 2] 18[1 1]	46	58968
1 2 3 4 7 16	1[3 3] 6[2 2] 13[1 1]	40	9828
1 2 3 7 8 28	1[3 3] 25[1 1]	52	9828
1 2 3 10 11 12	3[2 2] 22[1 1]	50	9828
1 2 3 10 11 14	2[2 2] 24[1 1]	52	9828

Table 68: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$				
Between	0.00	0.14	0.29	0.43	0.57
main	100.00				
main and 2fi	100.00				
2fi	13.79	75.86		10.34	
main and 3fi	10.71	78.57		10.71	
2fi and 3fi	100.00				
3fi	44.43	24.18	27.48	3.30	0.61

4.5 Foldover of the 36 Run Plackett-Burman Design

Table 69: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[18 18]	4	630
1 2 3	4[9 9]	8	7140
1 2 3 4	4[5 5] 4[4 4]	16	48195
1 2 3 9	4[6 6] 4[3 3]	16	10710
1 2 3 4 5	5[3 3] 10[2 2] 1[1 1]	32	118440
1 2 3 4 9	1[4 4] 6[3 3] 5[2 2] 4[1 1]	32	192780
1 2 3 4 15	3[4 4] 3[3 3] 6[2 2] 3[1 1]	30	50400
1 2 3 4 19	1[5 5] 1[4 4] 6[3 3] 3[2 2] 3[1 1]	28	13860
1 2 3 9 15	1[6 6] 10[3 3]	22	1512

Table 70: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.11	0.22	0.33	0.44	0.67
main	100.00					
main and 2fi	100.00					
2fi	10.81	72.97		16.22		
main and 3fi	8.33	75.00		16.67		
2fi and 3fi	100.00					
3fi	41.83	18.15	33.89	4.03	2.08	0.02

4.6 Foldover of the 44 Run Plackett-Burman Design

Table 71: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[22 22]	4	946
1 2 3	4[11 11]	8	13244
1 2 3 4	4[6 6] 4[5 5]	16	102641
1 2 3 9	4[7 7] 4[4 4]	16	33110
1 2 3 4 5	1[4 4] 10[3 3] 5[2 2]	32	278124
1 2 3 4 6	4[4 4] 5[3 3] 6[2 2] 1[1 1]	32	370832
1 2 3 4 9	1[5 5] 3[4 4] 6[3 3] 3[2 2] 3[1 1]	32	357588
1 2 3 5 14	2[5 5] 3[4 4] 3[3 3] 6[2 2] 1[1 1]	30	79464

Table 72: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$				
Between	0.00	0.09	0.18	0.27	0.36
main	100.00				
main and 2fi	100.00				
2fi	8.89	68.89		22.22	
main and 3fi	6.82	70.45		22.73	
2fi and 3fi	100.00				
3fi	40.65	14.05	35.09	4.53	4.68

4.7 Foldover of the 48 Run Plackett-Burman Design

Table 73: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[24 24]	4	1128
1 2 3	4[12 12]	8	17296
1 2 3 4	8[6 6]	16	77832
1 2 3 5	4[7 7] 4[5 5]	16	103776
1 2 3 36	4[8 8] 4[4 4]	16	12972
1 2 3 4 5	4[4 4] 8[3 3] 4[2 2]	32	674544
1 2 3 4 6	1[5 5] 4[4 4] 6[3 3] 4[2 2] 1[1 1]	32	518880
1 2 3 4 13	8[4 4] 8[2 2]	32	51888
1 2 3 4 24	2[5 5] 4[4 4] 4[3 3] 4[2 2] 2[1 1]	32	311328
1 2 3 4 27	1[6 6] 2[5 5] 3[4 4] 4[3 3] 3[2 2] 2[1 1]	30	51888
1 2 3 5 26	4[5 5] 1[4 4] 4[3 3] 6[2 2]	30	51888
1 2 3 6 8	1[6 6] 6[4 4] 4[3 3] 1[2 2] 4[1 1]	32	51888

Table 74: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$			
Between	0.00	0.17	0.33	0.50
main	100.00			
main and 2fi	100.00			
2fi	44.90	48.98	6.12	
main and 3fi	43.75	50.00	6.25	
2fi and 3fi	100.00			
3fi	46.22	46.97	6.70	0.12

4.8 Foldover of the 52 Run Plackett-Burman Design

Table 75: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[26 26]	4	1326
1 2 3	4[13 13]	8	22100
1 2 3 4	4[7 7] 4[6 6]	16	213200
1 2 3 52	4[12 12] 4[1 1]	16	325
1 2 4 7	4[8 8] 4[5 5]	16	46800
1 2 4 17	4[9 9] 4[4 4]	16	10400
1 2 3 4 5	1[7 7] 6[6 6] 1[5 5] 4[1 1]	24	15600
1 2 3 4 6	1[5 5] 6[4 4] 5[3 3] 4[2 2]	32	897000
1 2 3 4 12	5[4 4] 10[3 3] 1[2 2]	32	748800
1 2 3 4 16	4[5 5] 4[4 4] 1[3 3] 6[2 2] 1[1 1]	32	239200
1 2 4 6 9	1[6 6] 1[5 5] 6[4 4] 3[3 3] 3[2 2] 2[1 1]	32	83200
1 2 4 6 17	1[6 6] 3[5 5] 3[4 4] 4[3 3] 2[2 2] 3[1 1]	32	124800
1 2 4 6 25	3[5 5] 3[4 4] 6[3 3] 3[2 2] 1[1 1]	32	374400
1 2 4 9 17	2[6 6] 2[5 5] 3[4 4] 3[3 3] 4[2 2] 1[1 1]	30	93600
1 2 4 14 17	1[7 7] 1[6 6] 2[5 5] 4[4 4] 2[3 3] 3[2 2] 1[1 1]	28	15600
1 3 5 7 9	1[7 7] 10[4 4] 5[1 1]	32	1560
1 3 5 15 17	1[8 8] 3[5 5] 6[4 4] 1[3 3] 2[1 1]	26	5200

Table 76: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$				
Between	0.00	0.08	0.23	0.38	0.85
main	100.00				
main and 2fi	100.00				
2fi	7.55	72.81	15.98	3.55	0.11
main and 3fi	5.77	74.21	16.29	3.62	0.11
2fi and 3fi	100.00				

4.9 Foldover of the 60 Run Plackett-Burman Design

Table 77: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[30 30]	4	1770
1 2 3	4[15 15]	8	34220
1 2 3 4	4[8 8] 4[7 7]	16	307980
1 2 3 7	4[9 9] 4[6 6]	16	179655
1 2 3 4 5	1[5 5] 10[4 4] 5[3 3]	32	513300
1 2 3 4 6	1[6 6] 3[5 5] 6[4 4] 3[3 3] 3[2 2]	32	2053200
1 2 3 4 7	4[5 5] 5[4 4] 6[3 3] 1[2 2]	32	1539900
1 2 3 4 8	1[7 7] 4[5 5] 6[4 4] 4[2 2] 1[1 1]	32	307980
1 2 3 4 11	2[6 6] 3[5 5] 3[4 4] 6[3 3] 1[2 2] 1[1 1]	32	1026600
1 2 3 8 39	5[6 6] 10[3 3]	30	20532

Table 78: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$		
Between	0.00	0.07	0.20
main	100.00		
main and 2fi	100.00		
2fi	6.56	59.02	34.43
main and 3fi	5.00	60.00	35.00
2fi and 3fi	100.00		

4.10 Foldover of the 68 Run Plackett-Burman Design

Table 79: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[34 34]	4	2278
1 2 3	4[17 17]	8	50116
1 2 3 4	4[9 9] 4[8 8]	16	488631
1 2 3 7	4[10 10] 4[7 7]	16	313225
1 2 3 44	4[11 11] 4[6 6]	16	12529
1 2 3 4 5	5[5 5] 10[4 4] 1[3 3]	32	751740
1 2 3 4 6	1[6 6] 6[5 5] 5[4 4] 4[3 3]	32	2706264
1 2 3 4 8	1[7 7] 4[6 6] 6[4 4] 4[3 3] 1[1 1]	32	601392
1 2 3 4 9	3[6 6] 3[5 5] 6[4 4] 3[3 3] 1[2 2]	32	3006960
1 2 3 4 20	1[7 7] 1[6 6] 6[5 5] 3[4 4] 3[3 3] 2[2 2]	32	2555916
1 2 3 4 34	1[7 7] 3[6 6] 3[5 5] 4[4 4] 2[3 3] 3[2 2]	32	200464
1 2 3 4 45	2[7 7] 2[6 6] 3[5 5] 3[4 4] 4[3 3] 1[2 2] 1[1 1]	32	300696
1 2 3 6 44	4[6 6] 4[5 5] 1[4 4] 6[3 3] 1[2 2]	32	150348
1 2 3 7 15	1[8 8] 3[6 6] 4[5 5] 3[4 4] 1[3 3] 3[2 2] 1[1 1]	32	150348

Table 80: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.06	0.18	0.29
main	100.00			
main and 2fi	100.00			
2fi	5.80	56.52	36.23	1.45
main and 3fi	4.41	57.35	36.76	1.47
2fi and 3fi	100.00			

5 Half-Fraction of the Plackett-Burman Design

5.1 Half-Fraction of the 12 Run Plackett-Burman Design

Table 81: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[2 2] 1[1 1]	4	45
1 2 3	1[2 1] 3[1 0]	5	60
1 2 6	3[1 1]	6	60
1 2 3 4	1[1 1] 4[1 0]	6	180
1 2 3 5	1[2 0] 4[1 0]	5	15
1 2 6 9	3[1 1]	6	15

Table 82: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$			
Between	0.00	0.33	0.67	1.00
main		100.00		
main and 2fi	60.00		40.00	
2fi		95.45		4.55
main and 3fi		95.00		5.00
2fi and 3fi	63.33		36.67	

Table 83: Unique runs when projected into 3 dimensions.

n	Columns
8	None
6	1 2 6 9
5	1:10

5.2 Half-Fraction of the 20 Run Plackett-Burman Design

Table 84: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[3 3] 1[2 2]	4	144
1 12	1[4 4] 1[1 1]	4	9
1 2 3	1[2 2] 3[1 1]	8	288
1 2 4	3[2 1] 1[1 0]	7	336
1 2 12	2[2 2] 1[1 1]	6	72
1 2 13	1[3 1] 3[2 0]	5	48
1 2 16	1[3 2] 1[2 1] 2[1 0]	6	72
1 2 3 4	3[1 1] 4[1 0]	10	801
1 2 3 5	5[1 1]	10	153
1 2 3 6	1[2 1] 2[1 1] 3[1 0]	9	873
1 2 3 8	2[2 0] 1[1 1] 4[1 0]	8	414
1 2 3 13	1[2 1] 7[1 0]	9	162
1 2 4 12	2[2 1] 1[1 1] 2[1 0]	8	126
1 2 4 18	1[2 0] 2[1 1] 4[1 0]	9	216
1 2 5 16	1[2 2] 1[1 1] 4[1 0]	8	27
1 2 6 12	1[2 1] 2[2 0] 3[1 0]	7	126
1 2 7 8	1[2 1] 1[2 0] 1[1 1] 3[1 0]	8	90
1 2 16 18	2[2 1] 1[2 0] 2[1 0]	7	36
1 3 5 12	1[2 2] 1[2 1] 3[1 0]	7	9
1 3 6 16	1[2 2] 3[1 1]	8	18
1 4 6 15	4[2 0] 1[1 1]	6	9

Table 85: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.20	0.40	0.60	0.80	1.00
main		94.12		5.88		
main and 2fi	50.33		44.44		5.23	
2fi		83.44		16.10		0.46
main and 3fi		82.84		16.67		0.49
2fi and 3fi	50.98		44.69		4.33	

Table 86: Unique runs when projected into 3 dimensions.

n	Columns
8	1 2 5 17
7	1:3 7 9 11 15
6	1:5 7 10 12
5	1:18

5.3 Half-Fraction of the 24 Run Plackett-Burman Design

Table 87: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[3 3]	4	132
1 4	1[4 4] 1[2 2]	4	99
1 2 3	4[2 1]	8	264
1 2 4	2[2 2] 2[1 1]	8	528
1 2 5	2[3 1] 2[2 0]	6	132
1 2 10	1[3 2] 2[2 1] 1[1 0]	7	528
1 4 17	3[2 2]	6	44
1 4 20	1[3 3] 3[1 1]	8	44
1 2 3 4	1[2 1] 3[1 1] 3[1 0]	11	1914
1 2 3 5	1[2 1] 2[2 0] 1[1 1] 3[1 0]	9	1276
1 2 3 6	2[2 0] 2[1 1] 4[1 0]	10	440
1 2 3 9	4[1 1] 4[1 0]	12	396
1 2 3 16	1[2 0] 3[1 1] 4[1 0]	11	165
1 2 3 18	3[2 1] 1[2 0] 1[1 0]	8	44
1 2 4 7	6[1 1]	12	253
1 2 4 8	2[2 1] 6[1 0]	10	363
1 2 4 13	1[2 2] 1[2 1] 1[1 1] 3[1 0]	9	220
1 2 4 17	2[2 1] 2[1 1] 2[1 0]	10	836
1 2 5 6	2[3 0] 6[1 0]	8	33
1 2 5 20	2[2 1] 2[2 0] 2[1 0]	8	154
1 2 7 17	1[2 2] 2[1 1] 4[1 0]	10	286
1 2 10 14	1[2 1] 1[2 0] 2[1 1] 3[1 0]	10	440
1 2 10 18	1[3 1] 2[1 1] 4[1 0]	10	110
1 2 10 20	2[2 1] 1[2 0] 1[1 1] 2[1 0]	9	198
1 2 11 13	1[2 2] 4[1 1]	10	55
1 3 14 17	4[2 0] 2[1 1]	8	55
1 4 11 15	1[3 0] 2[2 0] 1[1 1] 3[1 0]	8	22
1 4 15 17	1[2 2] 2[2 1] 2[1 0]	8	22
1 11 15 20	1[3 1] 1[2 0] 1[1 1] 4[1 0]	9	22
5 7 11 21	1[2 2] 1[2 0] 1[1 1] 4[1 0]	9	11

Table 88: Correlation structure (% cases).

Between	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$ 0.00	0.33	0.67
main	57.14	42.86	
main and 2fi	45.45	46.75	7.79
2fi	45.22	48.82	5.96
main and 3fi	44.68	49.09	6.23
2fi and 3fi	45.12	48.31	6.57

Table 89: Unique runs when projected into 3 dimensions.

n	Columns
8	1:3 9 16
7	1 2 4 13:16 20
6	1:22

5.4 Half-Fraction of the 28 Run Plackett-Burman Design

Table 90: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[4 4] 1[3 3]	4	286
1 12	1[5 5] 1[2 2]	4	39
1 2 3	3[2 2] 1[1 1]	8	663
1 2 4	1[3 2] 3[2 1]	8	858
1 2 6	1[3 3] 1[2 2] 2[1 1]	8	312
1 2 8	3[3 1] 1[2 0]	7	208
1 2 12	2[3 2] 1[2 1] 1[1 0]	7	390
1 3 10	1[4 1] 3[3 0]	5	13
1 3 12	1[4 3] 2[2 1] 1[1 0]	7	39
1 5 26	1[4 2] 1[3 1] 2[2 0]	6	78
1 12 21	1[3 3] 2[2 2]	6	39

Table 91: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$							
Between	0.00	0.14	0.29	0.43	0.57	0.71	0.86	1.00
main		88.00		12.00				
main and 2fi	43.69		45.69		10.15		0.46	
2fi		73.93		24.15		1.85		0.07
main and 3fi		73.38		24.62		1.92		0.08
2fi and 3fi	42.99		47.40		9.35		0.26	

Table 92: Unique runs when projected into 3 dimensions.

n	Columns
8	1 3 4 6 22 24
7	1:5 7 9 13 15:18 21
6	1:9 11 12 14:16 18:21 25 26
5	1:26

5.5 Half-Fraction of the 36 Run Plackett-Burman Design

Table 93: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[5 5] 1[4 4]	4	459
1 7	1[6 6] 1[3 3]	4	102
1 2 3	2[3 3] 1[2 2] 1[1 1]	8	1128
1 2 4	3[3 2] 1[2 1]	8	1488
1 2 5	1[4 3] 1[3 2] 2[2 1]	8	1092
1 2 8	3[4 1] 1[3 0]	7	18
1 2 9	1[3 3] 3[2 2]	8	924
1 2 12	1[4 2] 3[3 1]	8	744
1 2 24	1[4 4] 2[2 2] 1[1 1]	8	108
1 3 7	2[4 2] 1[3 1] 1[2 0]	7	168
1 3 16	1[5 3] 2[3 1] 1[2 0]	7	96
1 4 17	1[5 2] 1[4 1] 2[3 0]	6	24
1 4 33	1[4 3] 2[3 2] 1[1 0]	7	138
1 7 11	3[3 3]	6	32
2 5 15	1[6 3] 3[3 0]	5	24

Table 94: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$									
Between	0.00	0.11	0.22	0.33	0.44	0.56	0.67	0.78	0.89	1.00
main		81.82		18.18						
main and 2fi	40.36		42.75		15.85		1.04			
2fi		67.13		28.19		4.51		0.15		0.01
main and 3fi		66.70		28.49		4.65		0.15		0.01
2fi and 3fi	37.86		47.39		13.60		1.13		0.02	

Table 95: Unique runs when projected into 3 dimensions.

n	Columns
8	1 2 4 10 13 14 16 18 20 22 23 28
7	1:4 6 7 9 12 14 18:20 23 25 31:34
6	1:4 6:9 11:14 16:19 21:24 26:29 31:34
5	1:34

5.6 Half-Fraction of the 44 Run Plackett-Burman Design

Table 96: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[6 6] 1[5 5]	4	651
1 7	1[7 7] 1[4 4]	4	210
1 2 3	1[4 3] 3[3 2]	8	2254
1 2 5	3[3 3] 1[2 2]	8	1659
1 2 7	1[5 4] 2[3 2] 1[2 1]	8	819
1 2 11	1[5 2] 3[4 1]	8	189
1 2 12	2[4 3] 1[3 2] 1[2 1]	8	2310
1 2 13	2[5 2] 1[4 1] 1[3 0]	7	126
1 2 16	3[4 2] 1[3 1]	8	784
1 2 19	1[4 4] 2[3 3] 1[1 1]	8	567
1 2 20	1[4 4] 1[3 3] 2[2 2]	8	1302
1 2 26	1[5 3] 2[4 2] 1[2 0]	7	252
1 3 12	1[5 3] 1[4 2] 2[3 1]	8	1134
1 10 34	1[5 5] 3[2 2]	8	42
1 12 34	3[4 3] 1[1 0]	7	42

Table 97: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$								
Between	0.00	0.09	0.18	0.27	0.36	0.45	0.55	0.64	0.73
main		75.61		24.39					
main and 2fi	34.38		45.01		18.00		2.61		
2fi		61.49		30.98		6.98		0.56	
main and 3fi		61.15		31.13		7.14		0.57	
2fi and 3fi	33.80		47.24		16.26		2.60		0.10

Table 98: Unique runs when projected into 3 dimensions.

n	Columns
8	1:4 8 15 17 20 22:24 28:29 32 34 42
7	1:42

5.7 Half-Fraction of the 48 Run Plackett-Burman Design

Table 99: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[6 6]	4	414
1 3	1[7 7] 1[5 5]	4	552
1 10	1[8 8] 1[4 4]	4	69
1 2 3	1[4 4] 2[3 3] 1[2 2]	8	3197
1 2 4	4[3 3]	8	299
1 2 5	1[5 4] 1[4 3] 1[3 2] 1[2 1]	8	1196
1 2 8	2[4 3] 2[3 2]	8	3864
1 2 11	1[5 3] 2[4 2] 1[3 1]	8	1932
1 2 15	2[5 3] 2[3 1]	8	138
1 2 26	2[4 4] 2[2 2]	8	414
1 2 27	2[5 2] 2[4 1]	8	276
1 2 31	4[4 2]	8	483
1 3 5	3[4 3] 1[2 1]	8	1012
1 3 8	1[5 4] 3[3 2]	8	1012
1 3 15	1[5 5] 1[3 3] 2[2 2]	8	276
1 3 26	2[5 3] 1[4 2] 1[2 0]	7	184
1 3 33	3[5 2] 1[3 0]	7	92
1 4 10	1[6 4] 2[4 2] 1[2 0]	7	23
1 4 32	1[6 2] 2[5 1] 1[4 0]	7	23
1 5 15	2[4 4] 1[3 3] 1[1 1]	8	276
1 6 10	1[6 4] 1[4 2] 2[3 1]	8	184
1 6 15	1[5 4] 2[4 3] 1[1 0]	7	46
1 10 15	1[6 5] 2[3 2] 1[2 1]	8	46
1 10 26	1[5 5] 2[3 3] 1[1 1]	8	23
1 10 28	1[6 3] 1[5 2] 1[4 1] 1[3 0]	7	92
1 18 20	1[6 3] 3[4 1]	8	92

Table 100: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$					
Between	0.00	0.17	0.33	0.50	0.67	0.83
main	40.00	53.33	6.67			
main and 2fi	32.61	45.22	18.55	3.48	0.14	
2fi	31.55	47.92	17.19	3.15	0.19	
main and 3fi	31.37	47.80	17.42	3.21	0.20	
2fi and 3fi	32.33	46.71	17.44	3.29	0.23	0.00

Table 101: Unique runs when projected into 3 dimensions.

n	Columns
8	1:4 8 11 14 15 19 21 23 24 27 28 38 39 41
7	1:46

5.8 Half-fraction of the 52 Run Plackett-Burman design

Table 102: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[7 7] 1[6 6]	4	960
1 50	1[12 12] 1[1 1]	4	1
2 5	1[9 9] 1[4 4]	4	48
2 11	1[8 8] 1[5 5]	4	216
1 2 3	1[7 1] 3[6 0]	5	24
1 2 4	3[5 2] 1[4 1]	8	416
1 2 5	1[5 5] 1[4 4] 2[2 2]	8	480
1 2 6	3[4 3] 1[3 2]	8	4320
1 2 11	1[5 4] 1[4 3] 2[3 2]	8	2832
1 2 13	1[4 4] 3[3 3]	8	2640
1 2 15	2[4 4] 1[3 3] 1[2 2]	8	1776
1 2 36	1[5 3] 3[4 2]	8	2096
1 2 50	1[7 6] 1[6 5] 2[1 0]	6	24
1 3 50	2[6 6] 1[1 1]	6	24
2 3 4	2[5 4] 1[3 2] 1[2 1]	8	672
2 3 50	3[6 1] 1[5 0]	7	24
2 4 17	1[5 4] 2[4 3] 1[2 1]	8	768
2 4 33	1[6 3] 2[5 2] 1[3 0]	7	96
2 4 43	1[6 5] 1[4 3] 1[3 2] 1[2 1]	8	384
2 5 12	1[6 4] 1[5 3] 1[4 2] 1[2 0]	7	240
2 5 17	1[7 4] 1[5 2] 1[4 1] 1[3 0]	7	48
2 5 18	1[5 5] 1[4 4] 1[3 3] 1[1 1]	8	192
2 5 19	1[6 4] 1[5 3] 2[3 1]	8	192
2 5 42	1[7 5] 2[4 2] 1[2 0]	7	24
2 5 49	1[6 5] 2[4 3] 1[1 0]	7	24
2 6 11	1[5 5] 2[3 3] 1[2 2]	8	528
2 6 23	2[5 3] 1[4 2] 1[3 1]	8	1248
2 7 39	1[6 3] 1[5 2] 2[4 1]	8	192
2 8 17	1[6 4] 2[4 2] 1[3 1]	8	336

Table 103: Correlation structure (% cases).

	$\|\frac{1}{n}\sum_k x_{ik}x_{jk}\|$												
Between	0.00	0.08	0.15	0.23	0.31	0.38	0.46	0.54	0.62	0.69	0.77	0.85	0.92
main		78.37		17.63		3.92						0.08	
main and 2fi	31.62		44.20		20.26		3.68				0.12		0.12
2fi		57.81		31.69		9.34		1.02		0.01		0.13	
main and 3fi		57.40		31.98		9.45		1.04		0.01		0.13	
2fi and 3fi	30.37		46.53		19.03		3.74		0.27		0.04		0.02

5.9 Half-Fraction of the 60 Run Plackett-Burman Design

Table 104: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[8 8] 1[7 7]	4	1044
1 6	1[9 9] 1[6 6]	4	609
1 2 3	1[5 4] 3[4 3]	8	3190
1 2 5	2[5 4] 1[4 3] 1[3 2]	8	6090
1 2 6	1[6 4] 1[5 3] 2[4 2]	8	3480
1 2 7	1[5 5] 2[4 4] 1[2 2]	8	2262
1 2 8	3[5 3] 1[4 2]	8	1740
1 2 11	1[6 5] 2[4 3] 1[3 2]	8	3480
1 2 14	1[5 5] 1[4 4] 2[3 3]	8	3480
1 2 21	3[4 4] 1[3 3]	8	2030
1 2 27	1[6 4] 2[5 3] 1[3 1]	8	1740
1 2 34	1[7 4] 2[5 2] 1[4 1]	8	522
1 2 35	2[6 3] 1[5 2] 1[4 1]	8	870
1 2 44	1[6 3] 3[5 2]	8	580
1 6 7	1[7 5] 3[4 2]	8	348
1 6 19	1[6 6] 3[3 3]	8	348
1 6 26	3[5 4] 1[2 1]	8	638
1 27 33	3[6 3] 1[3 0]	7	58

Table 105: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$											
Between	0.00	0.07	0.13	0.20	0.27	0.33	0.40	0.47	0.53	0.60	0.67	0.80
main		63.16		36.84								
main and 2fi	28.86		41.92		22.87		6.35					
2fi		52.64		34.13		11.16		1.97		0.09		
main and 3fi		52.46		34.09		11.36		2.00		0.09		
2fi and 3fi	29.20		44.31		20.21		5.53		0.71		0.04	0.00

Table 106: Unique runs when projected into 4 dimensions.

n	Columns
8	1:8 10:15 18:20 25 27 29 30 32 34 36 39:41 44: 48 51:53 56 58
7	1:58

5.10 Half-Fraction of the 68 Run Plackett-Burman Design

Table 107: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[9 9] 1[8 8]	4	1287
1 5	1[10 10] 1[7 7]	4	825
1 38	1[11 11] 1[6 6]	4	33
1 2 3	1[5 5] 3[4 4]	8	2310
1 2 4	3[5 4] 1[4 3]	8	4070
1 2 5	1[6 5] 2[5 4] 1[3 2]	8	5445
1 2 6	1[7 4] 1[6 3] 2[5 2]	8	1815
1 2 7	3[6 3] 1[5 2]	8	792
1 2 8	1[6 4] 3[5 3]	8	2508
1 2 9	2[5 5] 1[4 4] 1[3 3]	8	5247
1 2 10	1[6 5] 1[5 4] 2[4 3]	8	7656
1 2 16	2[6 4] 1[5 3] 1[4 2]	8	4092
1 2 21	1[6 6] 2[4 4] 1[3 3]	8	2772
1 2 36	1[7 5] 2[5 3] 1[4 2]	8	3498
1 2 38	2[6 5] 1[4 3] 1[3 2]	8	330
1 2 60	1[6 6] 1[5 5] 2[3 3]	8	264
1 3 23	2[7 4] 1[5 2] 1[4 1]	8	66
1 3 61	2[7 3] 1[6 2] 1[5 1]	8	132
1 4 38	1[7 5] 1[6 4] 2[4 2]	8	132
1 5 6	1[7 6] 3[4 3]	8	1089
1 5 12	3[5 5] 1[2 2]	8	561
1 5 16	1[8 6] 1[5 3] 2[4 2]	8	99
1 5 19	1[8 4] 2[6 2] 1[5 1]	8	99
1 5 38	2[6 5] 1[5 4] 1[2 1]	8	165
1 5 46	1[7 4] 2[6 3] 1[4 1]	8	924
1 5 56	3[6 4] 1[3 1]	8	561
1 6 27	1[7 5] 1[6 4] 1[5 3] 1[3 1]	8	264
1 6 38	1[6 6] 1[5 5] 1[4 4] 1[2 2]	8	231
1 11 25	1[7 3] 3[6 2]	8	44
1 11 62	1[7 6] 1[5 4] 1[4 3] 1[3 2]	8	396
1 16 38	1[7 7] 1[4 4] 2[3 3]	8	66
1 19 52	1[8 5] 1[6 3] 1[5 2] 1[4 1]	8	99
1 50 58	1[8 5] 3[5 2]	8	33

Table 108: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$												
Between	0.00	0.06	0.12	0.18	0.24	0.29	0.35	0.41	0.47	0.53	0.59	0.65	0.71
main		60.00		38.46		1.54							
main and 2fi	27.30		40.58		23.64		7.90		0.58				
2fi		49.91		34.04		12.90		2.85		0.29		0.00	
main and 3fi		49.76		33.97		13.08		2.89		0.30		0.00	
2fi and 3fi	27.43		42.93		21.55		6.74		1.22		0.11		0.00

Table 109: Unique runs when projected into 4 dimensions.

n	Columns
16	1:3 8 15 42
15	1:4 7 8 25 44 56
14	1:4 6 16 26 28 29 31 47 51 54 63 65
13	1:16 19 22 27 30 31 33 40 42 44 49 58 60 66
12	1:66

5.11 Half-Fraction of the 72 Run Plackett-Burman Design

Table 110: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[9 9]	4	840
1 3	1[11 11] 1[7 7]	4	420
1 4	1[10 10] 1[8 8]	4	1155
1 2 3	1[7 6] 2[5 4] 1[3 2]	8	1470
1 2 4	2[5 5] 2[4 4]	8	4410
1 2 5	4[5 4]	8	1540
1 2 6	1[6 6] 1[5 5] 1[4 4] 1[3 3]	8	4620
1 2 8	1[7 4] 2[6 3] 1[5 2]	8	1890
1 2 9	1[6 5] 2[5 4] 1[4 3]	8	11550
1 2 10	2[6 4] 2[5 3]	8	4935
1 2 14	1[7 5] 1[6 4] 1[5 3] 1[4 2]	8	4410
1 2 15	4[6 3]	8	420
1 2 22	2[7 4] 2[5 2]	8	630
1 2 35	2[6 5] 2[4 3]	8	3150
1 2 60	2[7 3] 2[6 2]	8	315
1 3 4	1[8 5] 1[6 3] 2[5 2]	8	840
1 3 8	1[8 4] 1[7 3] 1[6 2] 1[5 1]	8	210
1 3 9	1[6 6] 2[5 5] 1[2 2]	8	735
1 3 11	1[7 7] 2[4 4] 1[3 3]	8	630
1 3 13	1[8 6] 2[5 3] 1[4 2]	8	630
1 3 14	2[6 5] 1[5 4] 1[3 2]	8	2730
1 3 18	1[7 6] 1[5 4] 2[4 3]	8	2520
1 3 21	3[6 5] 1[2 1]	8	70
1 3 52	2[7 4] 1[6 3] 1[4 1]	8	630
1 3 65	1[7 5] 2[6 4] 1[3 1]	8	525
1 4 9	3[5 5] 1[3 3]	8	945
1 4 40	3[6 4] 1[4 2]	8	1750
1 4 47	1[7 5] 3[5 3]	8	630
1 4 51	1[6 6] 3[4 4]	8	2030
1 4 64	3[7 3] 1[5 1]	8	105
1 7 21	1[8 7] 3[4 3]	8	70
1 7 26	1[8 5] 2[6 3] 1[4 1]	8	210
1 8 47	1[8 4] 3[6 2]	8	140

Table 111: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$						
Between	0.00	0.11	0.22	0.33	0.44	0.56	0.67
main	34.78	47.83	17.39				
main and 2fi	26.58	40.99	22.86	8.20	1.37		
2fi	26.24	42.39	22.52	7.41	1.34	0.10	
main and 3fi	26.12	42.31	22.59	7.52	1.36	0.11	
2fi and 3fi	26.57	42.46	21.89	7.42	1.51	0.15	0.01

5.12 Half-Fraction of the 76 Run Plackett-Burman Design

Table 112: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[10 10] 1[9 9]	4	1980
1 74	1[18 18] 1[1 1]	4	1
2 9	1[11 11] 1[8 8]	4	432
2 13	1[12 12] 1[7 7]	4	288
1 2 3	1[10 1] 3[9 0]	5	36
1 2 4	3[5 5] 1[4 4]	8	6408
1 2 5	1[6 5] 3[5 4]	8	10584
1 2 8	3[6 4] 1[5 3]	8	6852
1 2 9	1[6 6] 1[5 5] 2[4 4]	8	3456
1 2 11	2[6 5] 1[5 4] 1[4 3]	8	5544
1 2 13	2[6 6] 1[4 4] 1[3 3]	8	2088
1 2 14	1[7 4] 3[6 3]	8	2676
1 2 45	1[7 6] 1[6 5] 2[4 3]	8	3888
1 2 74	1[10 9] 1[9 8] 2[1 0]	6	36
1 3 74	2[9 9] 1[1 1]	6	36
2 3 74	3[9 1] 1[8 0]	7	36
2 4 15	1[9 6] 2[6 3] 1[4 1]	8	216
2 4 17	2[7 4] 1[6 3] 1[5 2]	8	1800
2 4 19	1[7 5] 2[6 4] 1[4 2]	8	1332
2 4 21	1[7 6] 2[5 4] 1[4 3]	8	1836
2 4 23	2[7 5] 1[5 3] 1[4 2]	8	2808
2 4 25	1[7 7] 1[5 5] 1[4 4] 1[3 3]	8	1800
2 4 37	1[7 5] 1[6 4] 2[5 3]	8	3888
2 4 45	1[8 7] 2[5 4] 1[3 2]	8	792
2 4 55	1[7 7] 2[5 5] 1[2 2]	8	504
2 4 63	1[8 5] 1[7 4] 1[6 3] 1[4 1]	8	576
2 4 65	1[8 6] 2[6 4] 1[3 1]	8	504
2 5 64	1[8 5] 1[7 4] 2[5 2]	8	1008
2 6 25	1[8 5] 2[6 3] 1[5 2]	8	540
2 6 59	1[7 6] 1[6 5] 1[5 4] 1[3 2]	8	2880
2 8 23	1[8 6] 1[6 4] 1[5 3] 1[4 2]	8	1656
2 9 21	1[6 6] 2[5 5] 1[3 3]	8	1044

Table 113: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$												
Between	0.00	0.05	0.11	0.16	0.21	0.26	0.32	0.37	0.47	0.58	0.84	0.89	0.95
main		73.31		15.99		10.66						0.04	
main and 2fi	25.72		38.36		25.58		10.23				0.05		0.05
2fi		49.48		30.35		16.01		4.05	0.03	0.03		0.05	
main and 3fi		49.16		30.54		16.09		4.10	0.03	0.03		0.05	

5.13 Half-Fraction of the 80 Run Plackett-Burman Design

Table 114: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	2[10 10]	4	858
1 4	1[11 11] 1[9 9]	4	1560
1 8	1[12 12] 1[8 8]	4	585
1 2 3	2[6 5] 2[5 4]	8	7098
1 2 8	1[7 6] 1[6 5] 1[5 4] 1[4 3]	8	11076
1 2 9	1[6 6] 2[5 5] 1[4 4]	8	8385
1 2 10	2[7 5] 2[5 3]	8	975
1 2 11	1[8 5] 1[7 4] 1[6 3] 1[5 2]	8	3276
1 2 13	4[5 5]	8	429
1 2 18	1[7 5] 2[6 4] 1[5 3]	8	8892
1 2 26	2[7 4] 2[6 3]	8	1950
1 2 33	4[6 4]	8	1365
1 2 34	2[6 6] 2[4 4]	8	2496
1 2 70	1[8 6] 2[6 4] 1[4 2]	8	1287
1 3 8	1[7 7] 2[5 5] 1[3 3]	8	819
1 3 20	1[8 4] 2[7 3] 1[6 2]	8	819
1 4 6	1[7 6] 3[5 4]	8	4264
1 4 9	1[8 5] 3[6 3]	8	936
1 4 10	1[7 7] 1[5 5] 2[4 4]	8	2613
1 4 17	2[6 6] 1[5 5] 1[3 3]	8	2496
1 4 22	2[7 5] 1[6 4] 1[4 2]	8	3042
1 4 25	3[6 5] 1[4 3]	8	4264
1 4 32	1[8 6] 1[6 4] 2[5 3]	8	3042
1 4 45	1[7 6] 2[6 5] 1[3 2]	8	1872
1 4 47	3[7 4] 1[5 2]	8	936
1 7 49	2[8 4] 2[6 2]	8	117
1 8 11	1[8 7] 2[5 4] 1[4 3]	8	2106
1 8 17	1[9 6] 2[6 3] 1[5 2]	8	234
1 8 74	3[7 5] 1[3 1]	8	78
1 9 55	2[8 4] 1[7 3] 1[5 1]	8	234
1 12 25	1[9 5] 1[7 3] 2[6 2]	8	117
1 12 72	1[8 5] 2[7 4] 1[4 1]	8	468
1 29 30	3[6 6] 1[2 2]	8	156
1 29 51	1[10 6] 3[6 2]	8	78
1 29 74	1[8 8] 3[4 4]	8	117
1 35 72	1[9 7] 3[5 3]	8	39

Table 115: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$						
Between	0.00	0.10	0.20	0.30	0.40	0.50	0.60
main	28.57	51.95	19.48				
main and 2fi	24.99	39.29	23.98	9.99	1.75		
2fi	23.96	41.78	23.64	8.52	1.90	0.19	
main and 3fi	23.91	41.65	23.70	8.63	1.92	0.20	
2fi and 3fi	25.32	41.09	22.64	8.57	2.07	0.29	0.02

5.14 Half-Fraction of the 84 Run Plackett-Burman Design

Table 116: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[11 11] 1[10 10]	4	1722
1 3	1[12 12] 1[9 9]	4	1476
1 22	1[13 13] 1[8 8]	4	123
1 2 3	1[7 7] 2[5 5] 1[4 4]	8	6519
1 2 4	3[7 4] 1[6 3]	8	1435
1 2 5	1[7 6] 1[6 5] 2[5 4]	8	12218
1 2 7	2[7 5] 1[6 4] 1[5 3]	8	9471
1 2 8	3[6 5] 1[5 4]	8	4469
1 2 10	1[8 7] 1[6 5] 1[5 4] 1[4 3]	8	1681
1 2 11	1[8 5] 1[7 4] 2[6 3]	8	3936
1 2 12	2[6 6] 1[5 5] 1[4 4]	8	6273
1 2 15	1[7 6] 2[6 5] 1[4 3]	8	11070
1 2 16	1[8 6] 2[6 4] 1[5 3]	8	6560
1 2 21	1[7 5] 3[6 4]	8	2132
1 2 23	2[8 4] 1[7 3] 1[6 2]	8	410
1 2 27	1[8 4] 3[7 3]	8	287
1 2 36	1[6 6] 3[5 5]	8	3526
1 2 41	1[7 7] 1[6 6] 1[5 5] 1[3 3]	8	1476
1 2 50	1[8 6] 1[7 5] 1[6 4] 1[4 2]	8	820
1 3 7	1[8 5] 2[7 4] 1[5 2]	8	2624
1 3 11	1[9 6] 3[6 3]	8	1025
1 3 15	3[7 5] 1[4 2]	8	1681
1 3 19	1[8 7] 3[5 4]	8	2665
1 3 22	1[8 8] 1[5 5] 2[4 4]	8	451
1 3 50	2[7 6] 1[6 5] 1[3 2]	8	820
1 3 62	3[6 6] 1[3 3]	8	1845
1 3 66	1[9 6] 1[7 4] 1[6 3] 1[5 2]	8	615
1 4 22	2[7 6] 1[5 4] 1[4 3]	8	861
1 4 50	2[8 5] 1[6 3] 1[5 2]	8	451
1 5 35	3[8 4] 1[5 1]	8	164
1 5 61	2[8 5] 1[7 4] 1[4 1]	8	246
1 6 13	1[9 5] 2[7 3] 1[6 2]	8	615
1 6 40	1[8 6] 1[7 5] 2[5 3]	8	1107
1 10 22	1[9 7] 1[6 4] 2[5 3]	8	615
1 14 70	1[7 7] 1[6 6] 2[4 4]	8	205
1 22 27	1[8 7] 2[6 5] 1[3 2]	8	123
1 22 36	1[8 8] 2[5 5] 1[3 3]	8	123
1 27 63	3[8 3] 1[7 2]	8	41

Table 117: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$												
Between	0.00	0.05	0.10	0.14	0.19	0.24	0.29	0.33	0.38	0.43	0.48	0.52	0.62
main		51.85		44.44		3.70							
main and 2fi	24.93		37.35		24.66		11.38		1.63		0.05		
2fi		44.72		34.12		15.77		4.42		0.93		0.04	0.00
main and 3fi		44.63		34.00		15.91		4.48		0.94		0.04	0.00

5.15 Half-Fraction of the 100 Run Plackett-Burman Design

Table 118: Repeat and mirror-image patterns.

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
1 2	1[13 13] 1[12 12]	4	3456
1 98	1[24 24] 1[1 1]	4	1
2 5	1[15 15] 1[10 10]	4	576
2 7	1[14 14] 1[11 11]	4	576
2 19	1[16 16] 1[9 9]	4	144
1 2 3	1[13 1] 3[12 0]	5	48
1 2 4	3[8 5] 1[7 4]	8	9312
1 2 5	2[8 7] 1[6 5] 1[5 4]	8	9216
1 2 6	3[7 6] 1[6 5]	8	21280
1 2 7	1[8 7] 1[7 6] 2[6 5]	8	8832
1 2 9	1[7 7] 3[6 6]	8	13696
1 2 19	2[8 8] 1[5 5] 1[4 4]	8	1440
1 2 21	1[8 8] 1[7 7] 2[5 5]	8	6144
1 2 23	2[7 7] 1[6 6] 1[5 5]	8	6336
1 2 36	1[8 6] 3[7 5]	8	15072
1 2 98	1[13 12] 1[12 11] 2[1 0]	6	48
1 3 19	1[9 5] 3[8 4]	8	1824
1 3 98	2[12 12] 1[1 1]	6	48
2 3 18	1[9 8] 1[8 7] 2[5 4]	8	2208
2 3 98	3[12 1] 1[11 0]	7	48
2 4 7	1[9 7] 1[8 6] 1[7 5] 1[5 3]	8	3264
2 4 11	1[9 7] 2[7 5] 1[6 4]	8	1728
2 4 17	2[9 6] 1[7 4] 1[6 3]	8	3456
2 4 21	1[10 7] 2[8 5] 1[5 2]	8	1056
2 4 23	1[9 8] 1[8 7] 1[6 5] 1[4 3]	8	1152
2 4 25	1[9 7] 1[8 6] 2[6 4]	8	5760
2 4 31	1[9 6] 1[8 5] 2[7 4]	8	3840
2 4 37	1[10 7] 1[8 5] 1[7 4] 1[6 3]	8	2112
2 4 45	1[10 7] 1[9 6] 2[6 3]	8	960
2 4 49	1[8 8] 1[7 7] 1[6 6] 1[4 4]	8	3072
2 4 51	1[10 8] 1[8 6] 1[6 4] 1[5 3]	8	960
2 4 61	2[8 6] 1[7 5] 1[6 4]	8	5568
2 4 65	1[10 8] 2[7 5] 1[5 3]	8	1632
2 4 73	1[9 8] 1[7 6] 1[6 5] 1[5 4]	8	4800
2 4 81	1[10 9] 1[7 6] 1[6 5] 1[4 3]	8	1344
2 4 83	1[10 8] 1[8 6] 1[7 5] 1[4 2]	8	960
2 4 87	1[9 8] 2[7 6] 1[4 3]	8	2112
2 4 95	1[9 6] 2[8 5] 1[6 3]	8	1056
2 5 16	1[11 7] 2[8 4] 1[6 2]	8	192
2 5 19	1[11 8] 1[8 5] 1[7 4] 1[5 2]	8	384
2 5 36	1[10 6] 1[9 5] 1[8 4] 1[6 2]	8	576
2 5 43	1[9 9] 2[6 6] 1[4 4]	8	1344
2 6 53	1[9 9] 1[7 7] 1[6 6] 1[3 3]	8	768
2 6 79	1[10 7] 1[9 6] 1[7 4] 1[5 2]	8	576

Table 119: Repeat and mirror-image patterns (cont).

Columns	Repeat and mirror-image pattern	Unique runs	No. of projections
2 7 29	1[8 7] 2[7 6] 1[5 4]	8	2112
2 7 31	1[8 8] 2[6 6] 1[5 5]	8	1248
2 7 52	1[9 9] 1[7 7] 1[5 5] 1[4 4]	8	768
2 7 68	1[10 6] 2[8 4] 1[7 3]	8	192
2 7 84	2[9 5] 1[8 4] 1[7 3]	8	768
2 9 16	2[9 7] 1[6 4] 1[5 3]	8	1536
2 9 30	1[10 6] 1[9 5] 2[7 3]	8	768
2 18 51	1[10 9] 2[7 6] 1[3 2]	8	96
2 19 35	2[10 6] 1[7 3] 1[6 2]	8	192
2 19 51	1[10 10] 2[6 6] 1[3 3]	8	96
2 34 51	1[11 8] 2[8 5] 1[4 1]	8	48
2 51 83	1[12 8] 2[8 4] 1[5 1]	8	48

Table 120: Correlation structure (% cases).

	$\lvert\frac{1}{n}\sum_k x_{ik}x_{jk}\rvert$															
Between	0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.44	0.52	0.60	0.88	0.92	0.96
main		72.71		12.12		12.12		3.03							0.02	
main and 2fi	24.56		34.26		23.50		14.68		2.94					0.03		0.03
2fi		45.21		27.36		18.84		7.37		1.03	0.14	0.01	0.01		0.03	
main and 3fi		44.93		27.52		18.91		7.42		1.04	0.14	0.01	0.01		0.03	

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