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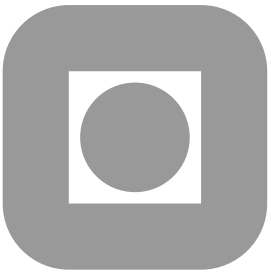

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STOCHASTIC RESERVOIR MODELS CONDITIONED TO NON-LINEAR PRODUCTION HISTORY OBSERVATIONS

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Abstract

Conditioning to production history observations is a hard problem since the observations are non-linearily related to the reservoir variables of interest. Moreover, computing the non-linear relation is extremely resource requiring. A formal stochastic approach for solving this problem is defined, and several sampling algorithms available for assessment of uncertainty are presented.

INTRODUCTION

The objective of reservoir evaluation is to decide on a reservoir management strategy. Predicting reservoir variables and forecasting of production under various depletion strategies are important aspects of this decision making effort. At the evaluation stage, all available information about the reservoir should be used in this process. Assessment of uncertainties will quantify the impact of this information.

The objective of stochastic reservoir evaluation is to provide such decision supporting information. In Omre and Tjelmeland (1997) a framework for this is defined, and in Hegstad and Omre (1998) the approach is demonstrated on a case study. The current paper is closely related to these two papers. The stochastic model used is presented on a graph, see Figure 1. This model is further explained in the paper, but a full definition can only be found in the references mentioned above. Several simulation algorithms available for sampling from the posterior model are presented and discussed.

AVAILABLE INFORMATION

The objective is to make predictions of reservoir variables and forecasts of reservoir production, and assess the associated uncertainties, based on the information available at the stage of evaluation. This information may be divided into two philosophically different types of information:

- General reservoir knowledge, including geological and physical understanding of the processes involved in the creation of the reservoir and the fluid flow during production. Empirical data from geologic analogues and comparable reservoirs may also be used.
- Reservoir specific observations, includes observations made in the particular reservoir under study. This will normally consist of seismic data, observations along well traces and production history.

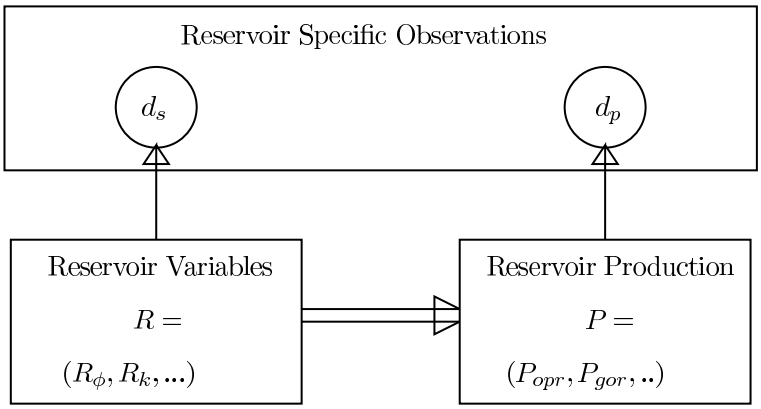

Figure 1: Graph of stochastic reservoir model.

Prior Model - General Reservoir Information

The prior model must be based on the general reservoir information. It is utmost important to be honest at this point and define a prior stochastic model that reflects the uncertainty about the reservoir under study based on the general reservoir information. This may entail that several scenarios for geologic origin of the reservoir under study is being defined. Hence reservoir geologists must be involved in defining the prior model. Each scenario should be assigned a probability, however, hence making it a stochastic model.

The prior stochastic model is represented by a joint pdf for the reservoir variables, R, and the reservoir production, P, ie. $f(r, p)$. The reservoir variables, r, is a multivariate, spatial variable discretized into a finite dimensional vector, containing spatial variables like pososity, permeability etc. The reservoir productionvariables, p, is a multivariate time series discretized into a finite dimensional vectors, containing the time series like oil-production-rate, gas-oil-ratio etc. in each well. This joint pdf is normally defined as a pdf on the reservoir variables R, $f(r)$; and a deterministic model for reservoir production given $R = r$ and the recovery strategy q, $p = v(r; q)$, termed $f(p|r)$. Computation of $p = v(r; q)$ is usually extremely resource requiring. The relation is represented by a double one-sided arrow on Figure 1.

The $f(r)$ model may be fully multivariately expressed as for Gaussian random fields; it may be expressed by local conditional pdfs as for Markov random fields; or it may only be available as a computer im-

plemented algorithm. Recall that R is very high-dimensional since it contain several reservoir variables defined on a large spatial grid, and so is P since it contain several production variables in several wells defined as a discretized time series.

Likelihood Model - Reservoir Specific Observations

The likelihood model links the variables of interest , namely reservoir variables and production, and the observations made, through forward models. The likelihood models for seismic data, well observations and production history are defined by the aquisition equipment and procedures. Hence geophysicists, petrophysicists and reservoir engineers must be involved in defining the respective likelihood models. A likelihood model specifies how likely it is to observe what is observed given the true reservoir variables and production.

The reservoir specific observations are normally of three types: seismic data, well observations and production history. In order to simplify notation only two classes are defined:

- Static data including seismic data and well observations, and termed d_s. The likelihood model is defined by:

$$[D_s|R = r] = g_s(r) + U_s \tag{1}$$

  with $g_s(r)$ being the forward model of the maesurement equipment being used to collect the static data; and the stochastic U_s being an error term representing modelling and observation error. The likelihood model is termed $f(d_s|r)$, and is represented by a single one-sided arrow on Figure 1.

- Dynamic production data termed d_p. The likelihood model is defined by:

$$[D_p|P = p] = g_p(p) + U_p \tag{2}$$

  with $g_p(p)$ being the sampling model of the production variables p; and the stochastic U_p being an associated error term. The sampling is normally done at certain points in time up to present time. The likelihood model is termed $f(d_p|p)$, and is represented by a single one-sided arrow on Figure 1.

The available information at time of evaluation consists of general reservoir knowledge and reservoir specific observations. The former is used in defining the prior pdf for the reservoir variables and production, $f(r, p)$. The latter must be linked to these variables of interest through likelihood models, $f(d_s|r)$ and $f(d_p|p)$, respectively. It is reasonable to assume that (D_s, D_p) are conditionally independent given $(R, P) = (r, p)$ since then they only depend on the approximation of the forward model and the observation error. Note further, that the joint model will not be Gaussian due to several non-linear functional dependencies.

RESERVOIR PREDICTIONS AND PRODUCTION FORECASTS - POSTERIOR MODEL

The stochastic model to be used for reservoir evaluation is displayed on the graph in Figure 1. The posterior pdf of interest is $f(r, p|d_s, d_p)$, ie. the stochastic model for reservoir variables and production given the available reservoir specific observations. By using Bayes relation and the assumption about conditional independence in the likelihood model, one has:

$$f(r, p|d_s, d_p) = \text{const} \times f(d_s|r) \times f(d_p|p) \times f(p|r) \times f(r) \tag{3}$$

This expression may be reformulated into:

$$f(r, p|d_s, d_p) = \text{const} \times f(d_p|p) \times f(p|r) \times f(r|d_s) \tag{4}$$

with $f(d_p|r) = \text{const} \times f(d_p|p) \times f(p|r)$. Assume that $f(r|d_s)$, ie. the reservoir variables conditioned to static data, is relatively simple to sample from. This can be obtained by using an algorithmic approach or the model in Eide at al. (1999) which results in a sequential algorithm. Alternatively sufficient linearity and Gaussianity assumptions can be made in order to make $f(r|d_s)$ Gaussian, which entails analytical tractability and availability of very fast algorithms. The resource requiring term is assumed to be $f(p|r)$ which includes the fluid flow simulations, $p = v(r; q)$. One major challenge is to define a simulation algorithm such that reliable samples from the posterior pdf, $f(r, p|d_s, d_p)$ can be efficiently obtained. In the next section various sampling algorithms available for this will be discussed.

Note further that if only a best prediction is required, ie. there is no need for assessing the uncertainty through sampling, the maximum posterior predictor (MAP) may be used. It is defined by:

$$(\hat{r}, \hat{p}) = \arg\max_{(r,p)} \{f(r, p|d_s, d_p)\} \tag{5}$$

which can be obtained by numerical optimization. In the case with all conditional pdfs being Gaussian and all functions of conditioning variables being linear, ie. the posterior being Gaussian, $(\hat{r}, \hat{p})$ will be identical to the Kriging predictor.

Simulation Algorithms

Four different classes of simulation algorithms will be discussed in this section. It is assumed that the pdf $f(r|d_s)$ is relatively simple to sample from. The difficult part is to condition on the production history, d_p, through $f(d_p|r) = \text{const} \times f(d_p|p) \times f(p|r)$ since the latter term involves the fluid flow model, $v(r; q)$, which is highly non-linear and computer resource requiring.

Rejection Sampling Algorithm

The algorithm is based on a simple proposal and acceptance/rejection concept, and can be considered a brute force approach. The algorithm is as follows:

Algorithm 1 *Rejection Sampling Algorithm*

Initiation:

- *Choose:* $\xi \geq \max_r\{f(d_p|r)\}$

Repeate:

- $r^\dagger \rightarrow f(r|d_s)$ - *proposal*
- $\alpha = \frac{f(d_p|r^\dagger)}{\xi}$ - *accept probability*
- *Prob* α : $(r_*, p_*) = (r^\dagger, v(r^\dagger; q))$ - *stop*

Then (r_*, p_*) is a sample from $f(r, p|d_s, d_p)$ being the pdf of interest. Note that normally the parameter ξ can be set equal to the mode value of the pdf of U_p in Expression 2. The algorithm is generally applicable, but low acceptance rates can in many cases make the algorithm unfeasible. The quality of the proposal is of course crucial. The fact that correct samples are obtained, and that the algorithm is simple to interpret and implement make it very attractive. The algorithm is applied in Hegstad and Omre (1997).

Sampling Importance Resampling Algorithm

The algorithm is based on sampling from a discretized approximation of the posterior pdf. The algorithm is as follows:

Algorithm 2 *SIR Algorithm*

Initiation:

- *Choose:* m - *discretization parameter*
- *Generate:*
  - $r_1, \cdots, r_m$ *iid* $f(r|d_s)$
  - $p_1 = v(r_1; q), \cdots, p_m = v(r_m; q)$
  - $\alpha_1 = f(d_p|p_1), \cdots, \alpha_m = f(d_p|p_m)$
  - $\bar{\alpha} = \sum_i^m \alpha_i$

Do:

- $(r_*, p_*) \rightarrow$ $Prob\{(r,p) = (r_i, p_i)\} = \frac{\alpha_i}{\bar{\alpha}}$; $i = 1, \cdots, m$

Then (r_*, p_*) is a sample from $\hat{f}(r, p|d_s, d_p)$ being a discrete approximation of $f(r, p|d_s, d_p)$. The approximation will approach the correct pdf as $m \rightarrow \infty$. The algorithm is generally applicable, but the approximation can in many cases be very rude, particularly for high-dimensional p. Many samples can of course be obtained from the same discrete approximation. The problem is however that for high-dimensional p one of the α may be completely dominant. Hence the same r will be sampled every time. The fact that approximate samples can be obtained fast by a procedure simple to interpret and implement make it attractive. The algorithm is described in larger detail in Rubin (1988).

Kitanidis-Oliver Algorithm

The algorithm is justified by the Gaussian case and is difficult to interpret for severe deviation from Gaussianity. Therefore it is assumed that both $f(r|d_s)$ and $f(d_p|p)$ are Gaussian in the presentation of the algorithm. Note, however, that the fact that reservoir variables and production are linked through the highly non-linear fluid flow model, $p = v(r; q)$, make the posterior pdf far from Gaussian.

Algorithm 3 *Kitanidis-Oliver Algorithm*

Do:

- $r^\dagger \rightarrow f(r|d_s, d_w) = Gauss(\mu_c, \Sigma_c)$ - *pivot sample*
- $d_p^\dagger \rightarrow Gauss(d_p, \Sigma_p)$ - *error sample*
- $(r_*, p_*) = \arg\min_{(r, p=v(r;q))}\{(r - r^\dagger)\Sigma_c^{-1}(r - r^\dagger)^T$
  $+ (g_p(p) - d_p^\dagger)\Sigma_p^{-1}(g_p(p) - d_p^\dagger)^T\}$

Then (r_*, p_*) is a sample from $\tilde{f}(r, p|d_s, d_p)$ being a pdf that approximates $f(r, p|d_s, d_p)$. In the case with $v(r; q)$ linear in r and $g_p(p)$ linear in p, ie. the posterior pdf being Gaussian, the algorithm can be shown to be correct. For non-linear $v(r; q)$ and $g_p(p)$ it is not known how reliable the algorithm is. Empirical testing presented in Omre at al. (2000) does indicate that the solutions r_* are too heterogenous in this case. Note that for the Gaussian case many other more efficient algorithms are available also.Note further that normally one global optimum does exist, but the optimization may be difficult due to r being high-dimensional and the object function possibly having multiple local optima. It is interesting, however, that the sampling problem can be phrased as an optimization problem, which may result in new interesting algorithms. The algorithm is applied in Kitanidis (1995) and He et al. (1997).

The Reduced Kitanidis-Oliver algorithm is defined in order to be able to perform the optimization in a reduced space. The algorithm is defined as follows:

Algorithm 4 *Reduced Kitanidis-Oliver Algorithm*

Initiate:

- *Choose:* $z = Br^T$ *- pilot point design matrix B;* $z \in \Re^m; r \in \Re^n; m \ll n$

Do:

- $r^\dagger \rightarrow f(r|d_s) = Gauss(\mu_c, \Sigma_c)$ *- pivot sample*
- $d_p^\dagger \rightarrow Gauss(d_p, \Sigma_p)$ *- error sample*
- $z^\dagger = Br^{\dagger T}$ *- initial pilot point values*
- $r(z) = r^\dagger + E\{C - \mu_c | B(C - \mu_c) = z - z^\dagger\}$
  - r parametrized by $z \in \Re^m$ *with C being* $Gauss(\mu_c, \Sigma_c)$.
- $\mathcal{A}_{B,r^\dagger} = \{r(z); \forall z \in \Re^m\}$ *- subspace of* $\Re^n$
- $(r_*, p_*) = \arg\min_{(r \in \mathcal{A}_{B,r^\dagger}, p = v(r;q))} \{(r - r^\dagger)\Sigma_c^{-1}(r - r^\dagger)^T$
  $+ (g_p(p) - d_p^\dagger)\Sigma_p^{-1}(g_p(p) - d_p^\dagger)^T\}$

Then (r_*, p_*) is a sample from $\tilde{\tilde{f}}(r, p|d_s, d_p)$ being a pdf that approximates $f(r, p|d_s, d_p)$. The reliability of the algorithm is not known. The fact that the optimization is performed in a subspace may not be so crucial since the dimensions of r are highly dependent. A too severe reduction in dimensionality may create too much heterogeneity in these dimensions, see Omre et al. (2000). The problems with multiple local optima in the object function also remain.

Two basic references for articles on production history matching, RamaRao et al. (1995) and Gómez-Hernández et al. (1997) , uses algorithms based on a pilot point consept very similar to the Reduced Kitanidis-Oliver algorithm. The algorithms in these papers do not randomize the object function by pivot and error samples, however, which gives the algorithms a Kriging flavor. The properties of their algorithms are not fully understood.

Markov Chain Monte Carlo Algorithms

This constitutes a class of algorithms widely used for sampling from complex, highly multivariate pdf's. One version of the algorithm is:

Algorithm 5 *McMC Algorithm*

Initiation:

- *Choose:*
  - $h(r|r^c)$ - *proposal pdf given* r^c
  - r_0 - *initial state,* $f(r_0|d_s, d_p) > 0$
  - $p_0 = v(r_0; q)$

Iterate $i = 1, 2, \cdots$

- $(r_i, p_i) = (r_{i-1}, p_{i-1})$
- $r^\dagger \rightarrow h(r|r_{i-1})$
- $p^\dagger = v(r^\dagger; q)$
- $\alpha = \min\{1, \frac{f(r^\dagger, p^\dagger|d_s, d_p)}{f(r_{i-1}, p_{i-1}|d_s, d_p)} \times \frac{h(r_{i-1}|r^\dagger)}{h(r^\dagger|r_{i-1})}\}$
- *Prob* α : $(r_i, p_i) = (r^\dagger, p^\dagger)$

Then $(r_*, p_*) = \lim_{i\rightarrow\infty}(r_i, p_i)$ is a sample from $f(r, p|d_s, d_p)$ being the pdf of interest. The class of McMC algorithms is generally applicable. It is iterative and will only converge towards a correct sample as $i \rightarrow \infty$. The convergence rate cannot be determined for complex problems, but the choice of proposal pdf, $h(\cdot|\cdot)$, and the procedure for generating the initial state, r_0, is known to be crucial for the convergence rate. So far the algorithm has not been widely used in practice, but the fact that it provides correct samples for general models make it very attractive. The algorithm is applied in Oliver et al. (1996) and Tjelmeland (1997). Promising generalizations of the McMC algorithm inspired by the production history conditioning problem are presented in Holden (1998) and Tjelmeland and Hegstad (1999).

Note further that the proposal pdf may be set equal to the pdf $f(r|d_s)$, hence independent of the previous iteration. Then the acceptance probability, α, will be dependent on $f(d_p|r)$ only. This special version is termed the independent sampling McMC algorithm and it may be useful when the prior model is only defined through an algorithm.

CONCLUSIONS

Sampling from the posterior pdf of reservoir variables and production given both static data and dynamic production data is a challenging task. The problem is not thoroughly understood, and hence it is important to state clearly the model assumptions made. The fact that fluid flow models are very computer requiring makes it critical to choose an efficient simulation algorithm. The practical problem is still unsolved.

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