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Failure Prediction from Condition Monitoring of Complex Systems*

by

Bo H. Lindqvist and Gunnhild H. Presthus

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Bo H. Lindqvist has homepage: http://www.math.ntnu.no/∼bo
E-mail: bo@math.ntnu.no
Address: Department of Mathematical Sciences, NTNU, N-7491 Trondheim,
Norway.

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Failure prediction from condition monitoring of complex systems

Bo H. Lindqvist * Gunnhild H. Presthus †

Abstract
We consider a technical system subjected to condition monitoring by a marker process $Y(t)$. Failure of the system is closely connected to the event that the process $Y(t)$ crosses a certain critical threshold. The monitored process $Y(t)$ depends probabilistically on the state of a latent process $S(t)$, representing the underlying technical condition of the system. The goal is to, based on the observation of the process $Y(t)$, estimate the distribution of the first passage time of $Y(t)$ of the critical threshold. The process $Y(t)$ is modeled as a piecewise Wiener process with change points determined by the latent process $S(t)$. A Bayesian approach involving Markov Chain Monte Carlo simulations is used for estimation.

Key Words: Wiener process, condition based maintenance, change point estimation.

1. Introduction

Any industrial production process involves maintenance as an integrated part. In fact, it has been reported that up to 70% of production costs can be attributed to maintenance. Efficient and cost-effective maintenance strategies are therefore sought. One solution is the so-called condition-based maintenance, in which (see definition in [Rausand and Høyland, 2004, p. 363]) maintenance actions are decided based on measurements of variables that are correlated with deterioration. The variables may be, e.g., temperature, pressure, erosion, vibration or noise levels. Condition-based maintenance requires a monitoring system for measurements of the variables, as well as a mathematical model that predicts the behavior of the deterioration process. When repair is difficult, involves risk, is costly in itself or leads to costly downtime, condition monitoring may be important to ensure that no production is lost. Examples that could be thought of are offshore structures, such as sub-sea structures or wind turbines. The motivating example of the present paper is from the latter field, where the object of study is the failure development of a wind turbine bearing. The bearing's temperature is then continuously monitored, assuming a Wiener-process type for its stochastic modeling.

Wiener processes have been used in a wide range of applications, perhaps mostly because of their tractable mathematical properties. In degradation modeling it is natural to consider the Wiener process with drift. In the case of a positive drift, it is well known that the first passage time of a Wiener process to a given level has the Inverse Gaussian distribution. In reliability engineering, this process has been studied in for example [Whitmore, 1986], where multiple modes of failure are represented by a multivariate Brownian motion. Length of stay in hospital has also been modeled by use of a Wiener processes by [Horrocks and Thompson, 2004], in a competing risks situation. Here the Wiener process represents a health level process and has multiple outcomes: death in hospital or healthy discarge. A time scale transformation is applied to a Wiener process in [Whitmore and Schenkelberg, 1997], with the aim of lifetime prediction. This time transformation is actually inspired by [Doksum and Høyland, 1992], see also Section 6. In

*Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway. Email: *bo@math.ntnu.no*

†Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, and Lloyd's Register Consulting, Sandvika, Norway. Email: *gunnhild.h.p@gmail.com*

[Whitmore et al., 1998] a bivariate Wiener process is connected to an unobservable marker process. Many different applications of Wiener processes are suggested, in a variety of fields, among them marriage failure, where the appropriate marker process is a social estrangement index, AIDS death, where CD4 cell count is the appropriate marker process, and metal fatigue failure, with dominant crack length as the marker process.

In the present paper we study Wiener processes with one or more change points, analyzed in a way inspired by [Shiryaev, 1963] and furthermore extending the previous study of [Lindqvist and Slimacek, 2013]. Whereas the latter authors developed estimates for the time of change points of a Wiener process, the present paper will examine the predicted distribution of the hitting time of a specified threshold. As already mentioned, in our application we use Wiener processes to model temperature in the bearing of a wind turbine, with distribution dynamically depending on an underlying unobserved failure development, modeled by a hidden Markov chain. A more detailed explanation of the case study is presented in Section 2. Some general theory for Wiener processes is displayed in Section 3, while Section 4 describes the basic model of the approach. The case of a single changepoint is studied in Section 5, which includes Bayesian statistical inference based on a simulated temperature process. The general case of m change points is briefly discussed in Section 6, which includes the analysis of simulated data for the case $m = 2$. Section 7 is the concluding section, which in particular discusses some possible extensions of the approach.

2. Failure development in a wind turbine bearing

The motivating example of this study is a wind turbine bearing which is continuously monitored. Following [Valland et al., 2012], the failure development in the wind turbine bearing can be described as a sequence of distinguished stages:

Stage 1: Impurities in oil

Stage 2: Mechanical wear

Stage 3: Micropitting

Stage 4: Chipping

Stage 5: Bearing break-down

Stage 6: Turbine shut-down

Let $Y(t)$ be the temperature at time t (days). Figure 1 from [Valland et al., 2012] shows an example of the temperature development in a wind turbine bearing through the six stages of failure development.

The temperature $Y(t)$ is an example of a marker process, i.e., a stochastic process generated by an individual under study which measures the "health" of the individual (see [Jewell and Kalbfleisch, 1996]). We will consider the marker process itself as the main object, being an example of condition monitoring as basis for condition-based maintenance.

Motivated by the example with a wind turbine bearing, the aim is to predict the time T until the process $Y(t)$ crosses a critical level. The time T is hence a first passage time. Assume that the process possesses some kind of stationarity under normal conditions, while under an emerging system failure, $Y(t)$ is expected to leave the stationary behavior and increase towards the critical border. In the wind turbine example, this means that the bearing enters state 1 and starts the failure development through the stages described above.

The marker process $Y(t)$ will be modeled as a stochastic process which depends on an unobservable stochastic process $S(t)$. The latter process represents the development through the failure stages as described earlier.

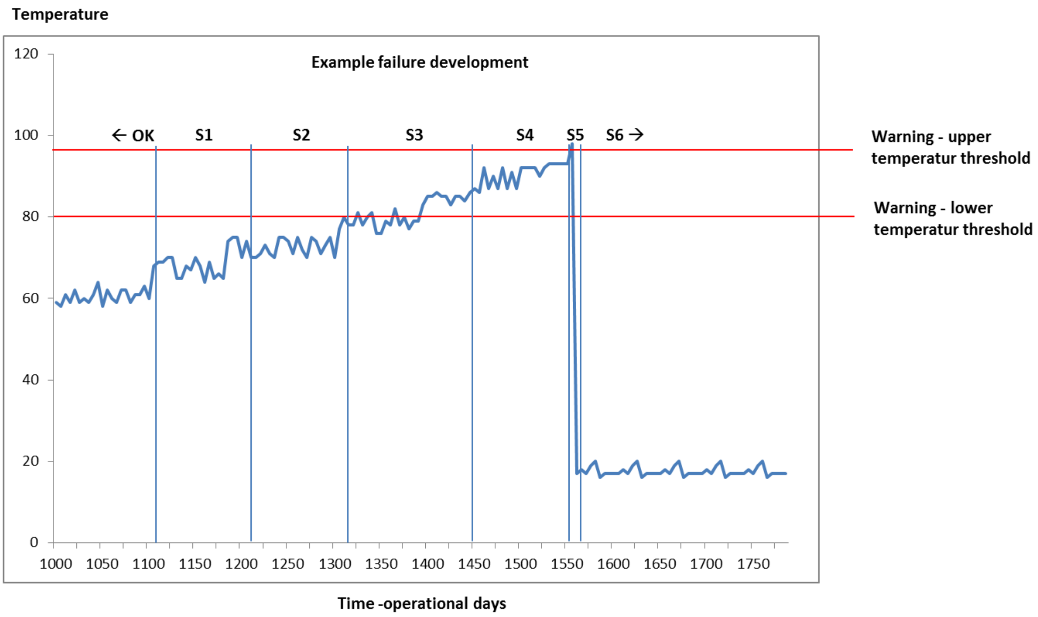

Figure 1: Example of the development of temperature, $Y(t)$, as a function of time t (days), in a wind turbine bearing through the six stages of failure development.

3. The Wiener process and the Inverse Gaussian distribution

Before presenting the model in the next section, we find it convenient to recall some definitions and properties of the Wiener process.

A stochastic process $W(t)$ is a Wiener process with drift coefficient ν and variance parameter σ^2 if

1. $W(0) = 0$ with probability one,
2. For every $t > 0$, $W(t)$ is normally distributed with mean νt and variance $\sigma^2 t$,
3. $W(t)$ has stationary and independent increments.

When $\nu = 0$, $W(t) = \sigma B(t)$, for a standard Brownian motion process $B(t)$. Note also that, given that $\nu > 0$, for times r and s such that $r > s$, $W(r)$ is first-order stochastically dominating $W(s)$, that is, for all x, $P(W(r) \geq x) \geq P(W(s) \geq x)$.

Define the first passage time T of a threshold value a as the first time the process crosses the threshold value:

$$T = \inf_{t>0}(W(t) > a).$$

An important and mathematically tractable attribute of the Wiener process, see, e.g, [Aalen and Gjessing, 2001], is that if $\nu > 0$, the first passage time to a level $W(t) \geq a > 0$ is Inverse Gaussian distributed, with density

$$f(t;\nu,\sigma,a) = \frac{a}{\sqrt{2\pi}\sigma} t^{-\frac{3}{2}} \exp\left(-\frac{(a-\nu t)^2}{2t\sigma^2}\right), \quad t > 0. \tag{1}$$

The mean and variance are given by

$$E[T] = \frac{a}{\nu}, \qquad Var[T] = \frac{a\,\sigma^2}{\nu^3}. \tag{2}$$

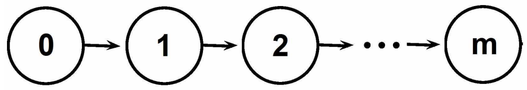

Figure 2: The stages $S(t)$ of the underlying failure development process is modeled as a Markov chain with state space $\{0, 1, \ldots, m\}$ and time-homogenous transition probabilities.

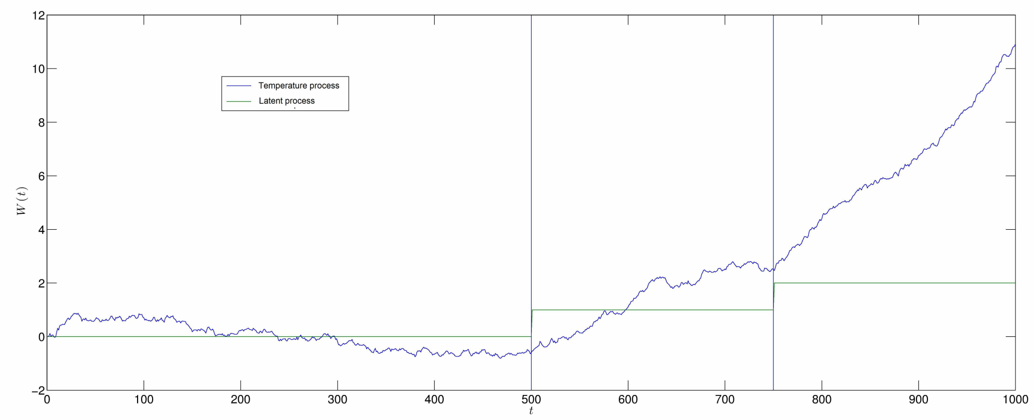

Figure 3: Simulation of the processes $S(t)$ and $Y(t)$ in the case of two switchpoints, $\tau_1 = 500$ and $\tau_2 = 750$.

It can be seen from (1) that the Inverse Gaussian distribution can be expressed by only two parameters $\mu = a/\nu$ and $\lambda = a^2/\sigma^2$. We shall however find it convenient to consider the distribution with three parameters, the drift parameter ν, variance parameter σ and threshold parameter a. The cumulative distribution $F(t; \nu, \sigma, a) = P(T \leq t)$ is then given by:

$$F(t; \nu, \sigma, a) = \Phi\left(\frac{\nu t - a}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2a\nu}{\sigma^2}\right)\Phi\left(\frac{-a - \nu t}{\sigma\sqrt{t}}\right), \tag{3}$$

and we say that T is Inverse Gaussian distributed with parameters ν, σ, a: $T \sim \mathcal{IG}(\nu, \sigma, a)$

4. Probabilistic modeling of the marker process $Y(t)$

Let the latent process $S(t)$ be modeled as a time-homogeneous Markov chain on the state space $\{0, 1, \ldots, m\}$, where $m = 6$ in our application to wind turbine bearing. It is assumed that $S(0) = 0$, and that the allowed transitions are only $0 \to 1 \to 2 \to \cdots$, see Figure 2. This means that the sojourn times in each state are exponentially distributed.

The observed temperature process $Y(t)$ is assumed to be a piecewise Wiener process where parameters will change when the stage $S(t)$ changes. More specifically, the parameters (ν, σ) of the Wiener process are defined as:

$$\begin{aligned} &= (0, \sigma_0) \quad \text{when } S(t) = 0 \text{ (“normal conditions”)}, \\ &= (\nu_i, \sigma_i) \quad \text{when } S(t) = i \in \{1, 2, \ldots, k\} \end{aligned}$$

where $0 < \nu_1 \leq \nu_2 \leq \ldots$.

Figure 3 shows a simulated process $Y(t)$ where $Y(0) = 0$, $k = 2$, and $S(t)$ switches from 0 to 1 at $\tau_1 = 500$ and from 1 to 2 at $\tau_2 = 750$.

5. Special case: One change point ($m = 1$)

Let τ be the time when the process $S(t)$ switches from 0 to 1. Thus, assume that $Y(t)$ follows a Wiener process with $Y(0) = 0$, and that until time τ, there is no drift, $\nu = 0$, while from time τ on, there is a positive drift ν. Assume that the variance parameter σ is the same in both stages.

Under these assumptions, let T be the time when $Y(t)$ crosses the level $a > 0$. Under the simplifying assumption that T is larger than τ, we thus have

$$T = \inf\{t > \tau | Y(t) \geq a\}.$$

Then conditional on τ and $Y(\tau)$, $T - \tau$ has an Inverse Gaussian distribution with drift parameter ν, variance parameter σ and threshold parameter $a - Y(\tau)$, that is $T - \tau \sim \mathcal{IG}(\nu, \sigma, a - Y(\tau))$.

5.1 Statistical inference with $m = 1$

We now describe the statistical inference problem, where it is convenient to use a Bayesian approach. For simplicity we furthermore consider a discretized time.

Thus assume that the latent process $S(t)$ as well as the marker process $Y(t)$ are observed at equidistant discrete time points t_i, where for simplicity we let $t_i = i, i = 1, 2, \ldots, n$. Now, the temperature increments $X_i \equiv Y(t_i) - Y(t_{i-1})$ are independent and normally distributed, with $X_i \sim N(0, \sigma_0^2)$ for $i = 1, 2, \ldots, \tau$; $X_i \sim N(\nu_1, \sigma_1^2)$ for $i = \tau + 1, \tau + 2, \ldots$.

Following [Shiryaev, 1963], we put a geometric prior on τ with parameter $q \in (0, 1)$,

$$\pi(\tau) = q(1 - q)^{\tau - 1};\ \tau = 1, 2, \ldots$$

This corresponds to assuming the transition probability $P_{01} = q$ for the (discrete time) Markov chain $S(t)$, that is, the probability of experiencing a switch from state 0 to state 1, on any time unit, is q. This may seem like a reasonable assumption for the wind turbine bearing example, where state 1 corresponds to the event of occurrence of impurities in the oil, which can be viewed as an externally triggered event, independent of the wear of the bearing. Note that the expected value of τ is $1/q$.

5.1.1 The posterior predictive distribution of T

Given observations $x_1, \ldots, x_n$ of the temperature increments $X_1, \ldots, X_n$, the problem is to predict the distribution of the time T of exceedance of level a.

At first, we consider ν and σ as known. [Lindqvist and Slimacek, 2013] noted that it may be a reasonable assumption that ν and σ are known from expert judgment or statistical analysis of past data. Now,

$$\begin{aligned} &P(T \leq t | x_1, \ldots, x_n; \nu, \sigma, a) \\ &= \sum_{\tau \in \Omega} P(T \leq t | \tau, x_1, \ldots, x_n; \nu, \sigma, a) \pi(\tau | x_1, \ldots, x_n; \nu, \sigma) \end{aligned} \tag{4}$$

where Ω is the range of τ.

For computation of the right hand side of (4), the first factor can be derived from properties of the Wiener process and Inverse Gaussian distribution, to be discussed below. For the second factor, the posterior of τ, there is no simple analytic expression, and a Monte Carlo simulation approach will be used.

5.1.2 *The posterior distribution of τ*

The likelihood function for the data is (see [Lindqvist and Slimacek, 2013])

$$L(\tau|x_1,\ldots,x_n) \propto \begin{cases} \exp\{-\frac{1}{2\sigma^2}(\sum_{i=1}^{\tau} x_i^2 + \sum_{i=\tau+1}^{n}(x_i-\nu)^2)\} & \text{if } \tau \le n \\ \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2\right\} & \text{if } \tau > n, \end{cases}$$

and the posterior for τ is proportional to $\pi(\tau)L(\tau|x_1,\ldots,x_n)$.

Multiplying L by $\exp\left\{(1/2\sigma^2)\sum_{i=1}^{n} x_i^2\right\}$ we get the posterior distribution $\pi(\tau|x_1,\ldots,x_n)$ for τ on the form

$$\propto q(1-q)^{\tau-1}\cdot \begin{cases} \exp\{\frac{1}{2\sigma^2}[2\nu\sum_{i=\tau+1}^{n} x_i - (n-\tau)\nu^2]\} & \text{if } \tau \le n \\ 1 & \text{if } \tau > n. \end{cases}$$

5.1.3 *The conditional distribution of T*

In order to compute the first factor on the right hand side of (4), $P(T \le t|\tau, x_1,\ldots,x_n;\nu,\sigma,a)$, we need to distinguish between the cases $\tau < n$ and $\tau \ge n$.

In the former case,

$$\begin{aligned} P(T \le t|\tau, x_1,\ldots,x_n;\nu,\sigma,a) &= P(T - t_n \le t - t_n|\tau, x_1,\ldots,x_n;\nu,\sigma,a)) \\ &= P(S \le t - t_n|\tau, x_1,\ldots,x_n;\nu,\sigma,a - Y(t_n))) \\ &= F(t-t_n; a - Y(t_n), \nu, \sigma), \end{aligned}$$

where $F(\cdot;\beta,\gamma,\delta)$ is the Inverse Gaussian cumulative distribution function given in equation (1) with threshold parameter β, drift parameter γ and variance parameter δ.

In the situation where $\tau > t_n$, because of the assumption that the threshold is not crossed before τ, $P(T < \tau) = 0$, the process should be shifted to the new point $(\tau, Y(\tau))$, from which the shifted threshold time is Inverse Gaussian, $S = T - \tau \sim \mathcal{IG}(\nu,\sigma,a - Y(\tau))$. However, $Y(\tau)$ is a future state and thus unknown. Conditioning on $Y(\tau)$ we have,

$$\begin{aligned} P(T \le t|\tau, Y(\tau), x_1,\ldots,x_n;\nu,\sigma,a) &= P(T - t_n \le t - t_n|\tau, Y(\tau), x_1,\ldots,x_n;\nu,\sigma,a)) \\ &= P(S \le t - t_n|\tau, Y(\tau), x_1,\ldots,x_n;\nu,\sigma,a - Y(\tau)) \\ &= F(t - t_n;\nu,\sigma,a - Y(\tau)). \end{aligned}$$

In order to uncondition on $Y(\tau)$, one may simulate from the distribution $N(0,(\tau-n)\sigma^2)$ of $Y(\tau) - Y(n)$, or use the rough approximation that $Y(\tau) \approx Y(n)$.

[Presthus, 2014] also considered an estimator of $P(T \le t|x_1,\ldots,x_n;\nu,\sigma,a)$ completely based on simulation. More precisely, this simulation approach simulates a large number of Wiener processes, starting at the last observation $Y(n)$, until the processes crosses the threshold value a.

5.1.4 *Simulated example*

Figure 4 shows a simulation from the temperature process. Table 1 shows some summary statistics of the posterior distribution of τ when the process is observed until, respectively, time $n = 1400$, $n = 1700$, $n = 3000$. It is seen that the uncertainty is fairly large for the two lowest observation times, while at time 3000, the change point is very well estimated, as should be expected.

The predictive distribution of T is shown in Figure 5, again for the observation times respectively $n = 1400, 1700, 3000$. The blue curve in the three plots is the posterior predictive distribution of T. It is seen that in the case $n = 3000$, the curve is very close to

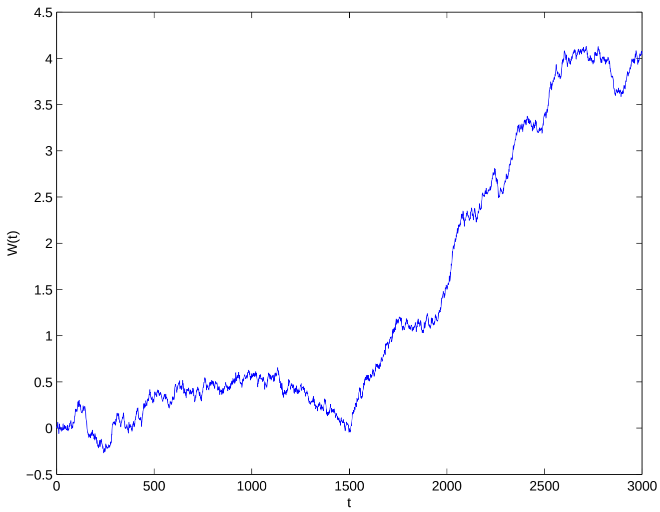

Figure 4: Piecewise Wiener process simulated in the time interval $[0, n] = [0, 3000]$, with one change point $\tau = 1500$. The drift parameter after the change point is $\nu = 0.003$, and variance parameter is the same for both pieces, $\sigma = 0.02$. The critical threshold is $a = 14$.

$n = 1400$	Mean	4 378
	Median	3 459
	95 % HPD	[1 292, 10 389]
$n = 1700$	Mean	3 352
	Median	2 149
	95 % HPD	[1 254, 9 138]
$n = 3000$	Mean	1 507
	Median	1 497
	95 % HPD	[1 292, 1 682]

Table 1: Estimated values of τ for observation periods of length n and prior distribution for τ being geometric with $q = 1/3000$. (True value of τ is $\tau = 1500$).

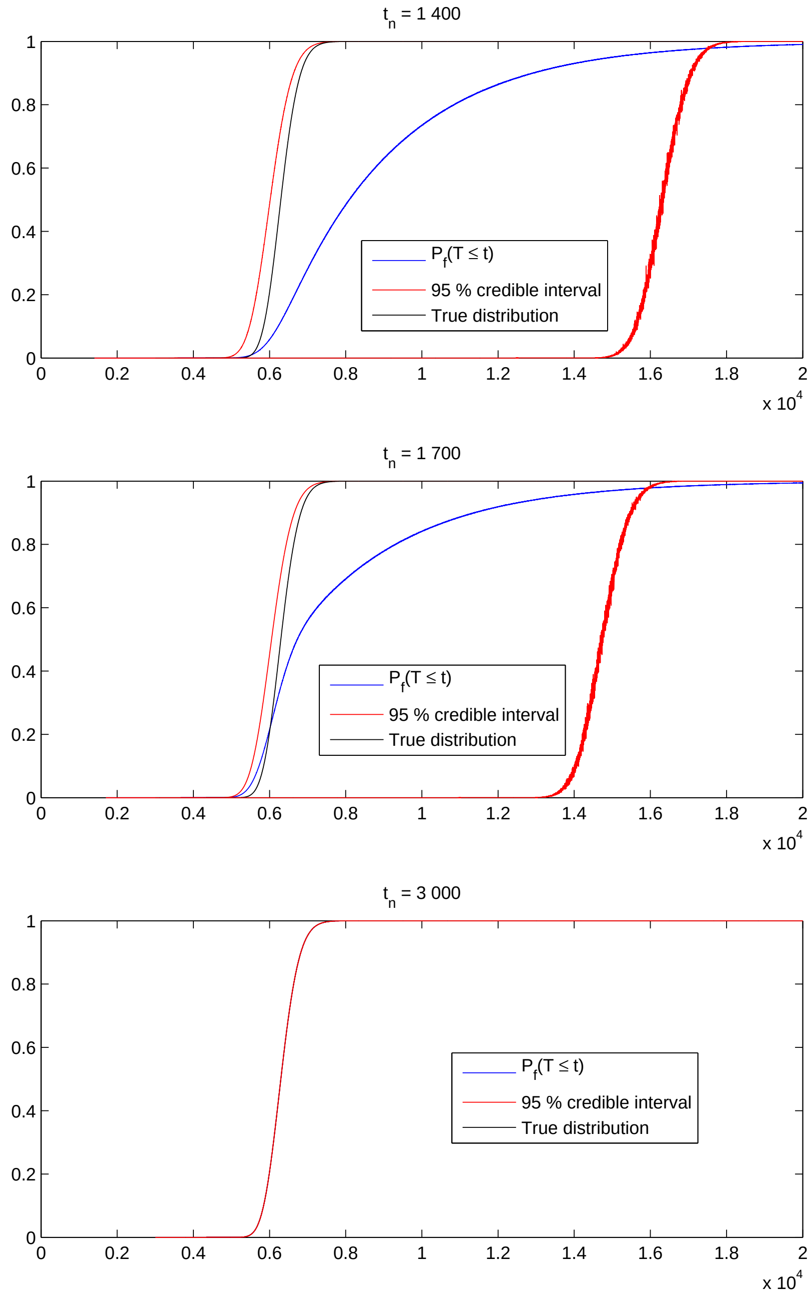

Figure 5: Estimated predictive distribution of T for observation periods of different lengths n and prior distribution for τ being geometric with $q = 1/3000$. (True value of τ is $\tau = 1500$).

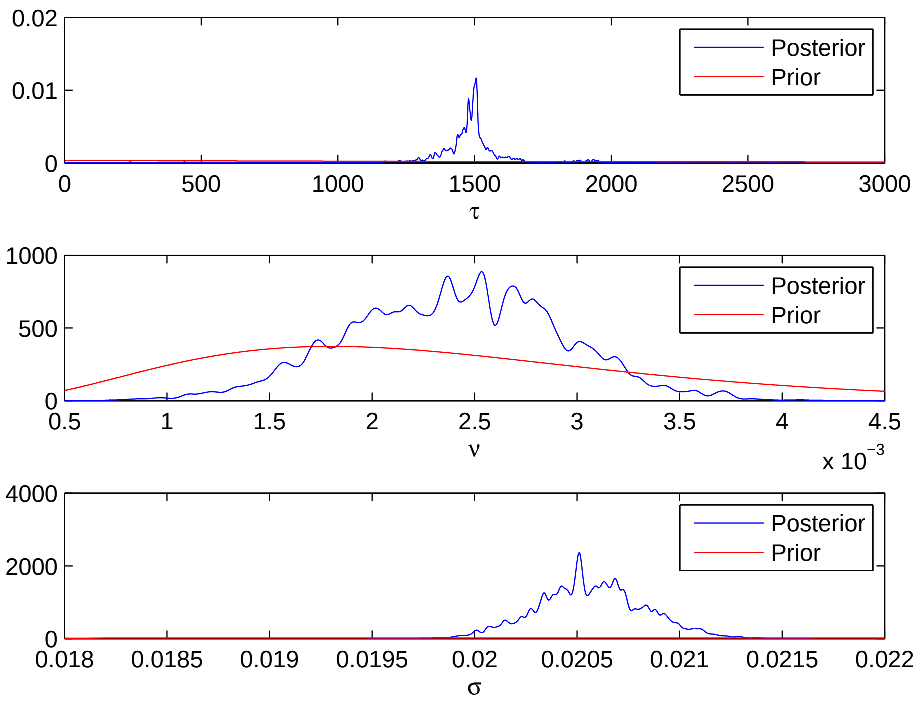

Figure 6: Priors and posteriors for unknown τ, ν, σ. Posteriors are for observation with $n = 3000$. Prior distributions: $\tau \sim$ geometric(1/3000); $\nu \sim$ gamma(4,0.0006); $\sigma \sim$ gamma(4,0.01). True values: $\tau = 1500$, $\nu = 0.003$, $\sigma = 0.02$.

the true distribution, which corresponds to knowing the value of τ. The curves forming the 95% credible intervals are based on the credibility intervals for τ as obtained from the respective posterior distributions.

[Presthus, 2014] also considered the case when τ, ν, σ are all unknown parameters, and performed a Bayesian analysis using properly tuned prior distribtuions and the Metropolis-Hastings method. Graphs of the posterior distributions are given in Figure 6. Note that the example is based on observation at time 3000, which implies that the variation in the posterior of τ is fairly low.

6. The general case: m change points

Consider now the setting when the state space of the underlying process $S(t)$ is $\{1, 2 \ldots, m\}$, in which case the marker process $Y(t)$ has m change points,

$$\tau_1 < \tau_2 < \cdots < \tau_m$$

and corresponding Wiener-process parameters

$$\begin{aligned} \vec{\nu} &= [0, \nu_1, \ldots, \nu_m] \\ \vec{\sigma} &= [\sigma_0, \sigma_1, \ldots, \sigma_m]. \end{aligned}$$

The likelihood for the increments $X_i = Y(i) - Y(i-1)$ is, as for the case $m = 1$, in principle straightforward to write down, but for a given observation time n it needs to be stated explicitly for each of the cases $n \in (\tau_j, \tau_{j+1}]$, $j = 1, \ldots, m-1$.

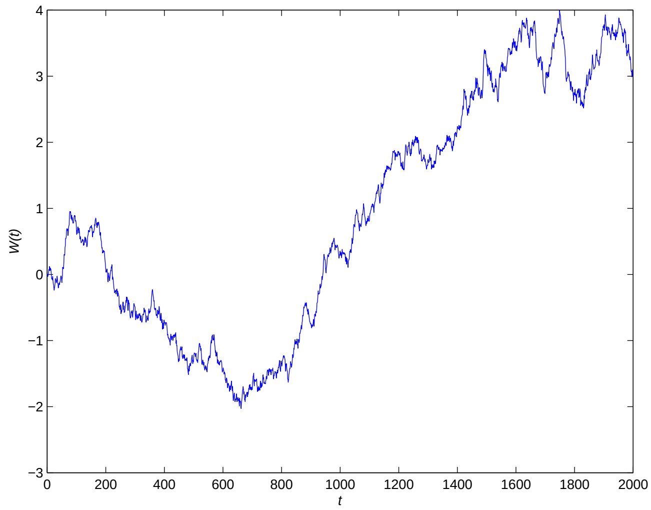

Figure 7: Simulated example: Two change-points. Piecewise Wiener process with two change points $\tau_1 = 800$ and $\tau_2 = 1400$ simulated in the time interval $[0, n] = [0, 2000]$. Parameter values: $\nu_0 = 0$, $\sigma_0 = 0.05$, $\nu_1 = 0.03$, $\sigma_1 = 0.05$, $\nu_2 = 0.039$, $\sigma_2 = 0.065$. Critical temperature threshold is $a = 13$.

In correspondence with the Markovian assumption for $S(t)$, the prior distribution for $(\tau_1, \ldots, \tau_m)$ should be chosen by giving $\tau_1, \tau_2 - \tau_1, \ldots, \tau_m - \tau_{m-1}$ independent exponential distributions in general, and geometric distributions in the discrete time case.

The computation of the posterior predictive distribution of T is similar to the one for the case with one change point, namely,

$$\begin{aligned} P(T \le t | x_1, \ldots, x_n; \vec{\nu}, \vec{\sigma}, a) = \\ \sum_{(\tau_1, \ldots, \tau_m) \in \Omega} P(T \le t | \tau_1, \ldots, \tau_m, x_1, \ldots, x_n; \vec{\nu}, \vec{\sigma}, a) \\ \times \pi(\tau_1, \ldots, \pi_m | x_1, \ldots, x_n; \nu, \sigma) \end{aligned}$$

where Ω is the range of $(\tau_1, \ldots, \tau_m)$, and the summation needs to be done by sampling vectors $(\tau_1, \ldots, \tau_m)$ from the joint posterior of the τ_i.

The computation of the terms

$$P(T \le t | \tau_1, \ldots, \tau_m, x_1, \ldots, x_n; \vec{\nu}, \vec{\sigma}, a)$$

involves numerical or Monte Carlo integrations in addition to analytic expressions from the Inverse Gaussian distribution. This extends the procedure we indicated above for the case $m = 1$. Some of the challenges for the computation for a general m are touched in the case $m = 2$, which is treated in detail in [Presthus, 2014].

Figure 7 shows a simulated example with $m = 2$, while Figure 8 gives the resulting estimates and confidence curves for observations at three different points in time.

6.1 Time-transformed Wiener process

Note that in the above presentation of the case with general m, we allow the variance parameters σ_j to vary freely. Below we indicate that by imposing a certain relation between

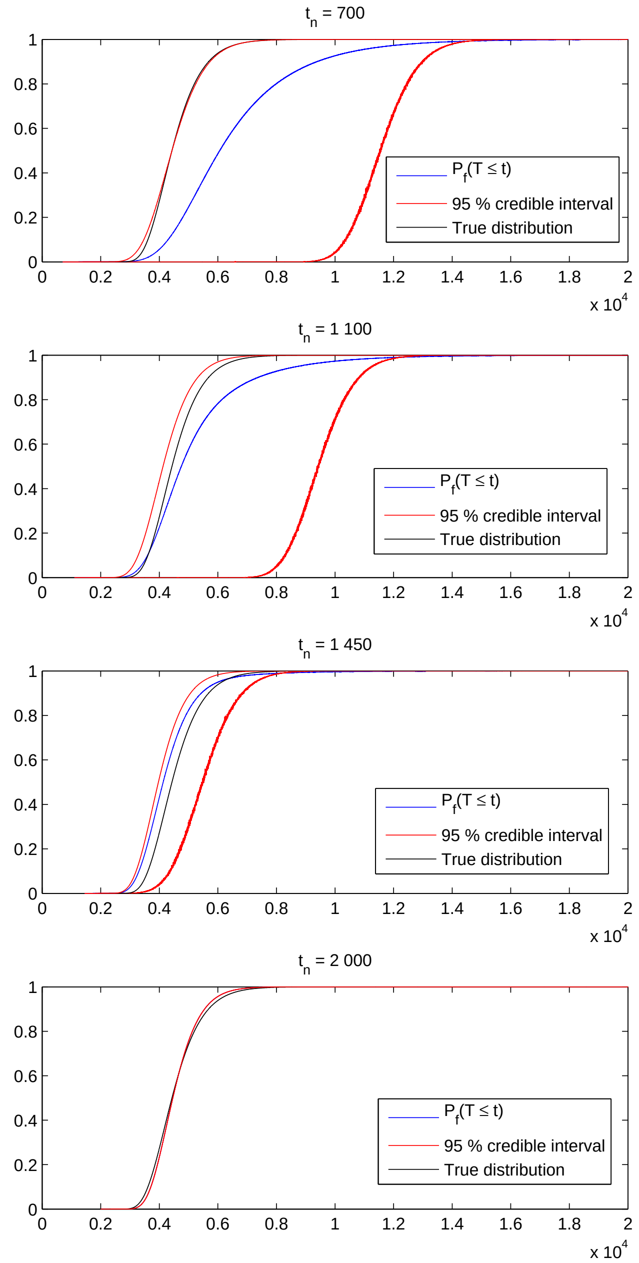

Figure 8: Estimated predictive distributions for two change-points, for observation periods of different lengths n and prior distribution for τ_1, τ_2 given by τ_1 geometrically distributed with probability $1/2000$, and $\tau_2 - \tau_1$ geometrically distributed with probability $1/1000$. (The true values are $\tau_1 = 800$ and $\tau_2 = 1400$).

the σ_j, the calculation of the posterior predictive distribution of T may be considerably simplified. The following Proposition from [Doksum and Høyland, 1992] is crucial:

Proposition. *Let $\xi(t)$ be a nonnegative strictly increasing and continuous function on $[0, \infty)$ with $\xi(0) = 0$ and let $\{W_0(t), t > 0\}$ be a Wiener process with drift $\eta > 0$ and variance parameter σ^2. Let T be the first time the process $W_0(\xi(t))$ hits the threshold $a > 0$. Then the cdf of T equals $F(\xi(t); \nu, \sigma, a)$ where F is given in (3).*

The result leads to a simplification of the calculations in the m change-point case by defining

$$\xi(t) = \begin{cases} t & \text{for } \tau_1 < t \leq \tau_2 \\ \tau_j + \frac{\nu_j}{\nu_{j-1}}(t - \tau_j) & \text{for } \tau_j < t \leq \tau_{j+1},\ j = 2, \dots \end{cases}$$

The drawback of such a simplified approach is, however, that the variance parameters are restricted by $\sigma_j = (\nu_j/\nu_{j-1})\sigma_{j-1}$ for $j = 2, \dots$ (see [Presthus, 2014] for further details).

7. Concluding remarks and further work

In the framework of [Valland et al., 2012], several health indicators of bearing failure were suggested. While we used the temperature as the only such indicator, we might have considered multi-dimensional processes, either of multivariate Wiener type, or combinations of Wiener-processes and other types of processes. A challenge would then be that the health indicators most probably are correlated.

Another suggestion for a model extension would be to include more than one failure mode, thus assuming a kind of competing risks situation. The type of failure considered in the present paper is a “soft” one, resulting from mechanical wear. One could also include the possibility of a shock, which would have a larger effect along the failure development.

For problems where the deterioration is cumulative, such as corrosion or crack growth, the gamma process might be a more suitable process. [Fouladirad et al., 2008] presented a frequentistic approach to estimating change points of such a deterioration process, exemplified by the gamma process.

Switching models like the one suggested in this paper are common in the literature. A reliability application is given by [Chiquet et al., 2008]. In a potential extension of the present work one might let the latent processes $S(t)$ be given as by a more general stochastically monotone Markov chains, with increasing paths relative to some partial order.

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