

# DIFTA

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## An example by J. Serrin.

**Abstract:** The right concept of solution is always a fundamental issue in the theory of partial differential equations. Some surprising features are, however, not mentioned in the textbooks.

An old example by J. Serrin exhibits a strange phenomenon, related to the definition of weak solutions of (uniformly) elliptic equations. The *a priori* integrability  $\nabla u \in L^2_{\text{loc}}$  is decisive even for linear equations in divergence form. The less demanding integrability assumption  $\nabla u \in L^{2-\varepsilon}$  for the gradient of the solution makes sense but allows solutions that are too weak. Such “false solutions” violate the maximum principle and so the uniqueness of the solution to the Dirichlet boundary problem is lost. The whole theory loses its familiar contours!

We will show the example and discuss some related results.