On the Boundary Control of Systems of Conservation Laws

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In this lecture, we want to discuss about the problem of the boundary controllability for systems of conservation laws. Consider a $n \times n$ system

$$u_t + f(u)_x = 0 \qquad t \geq 0, \quad x \in]a,b[\tag{1}$$

we study the properties of the reachable set $\mathcal{R}(T)$ (i.e. the set of all maps $\varphi \in L^1([a,b])$ such that there exists $u = u(t,x)$ entropic solution of (1) such that $u(T,\cdot) = \varphi$). Under the standard assumptions that (1) is strictly hyperbolic, all the families are genuinely nonlinear or linearly degenerate in the sense of Lax and the eigenvalues of Df are bounded away from zero (i.e. the boundaries are not characteristics), we obtain the following two results.

I) Asymptotic stabilizability of the constant states. Consider a constant u^* and an initial datum $\bar{u} \in BV([a,b])$ with small total variation and near u^*, in the L^1-norm, we show the existence of a solution $u = u(t,x)$ of (1) such that

$$u(0,\cdot) = \bar{u}, \qquad \text{Tot.Var.}\{u(t)\} \leq C\, e^{-2^{\kappa t}},$$

$$\int_a^b \big|u(t,x) - u^*\big|\, dx \leq C\, e^{-2^{\kappa t}},$$

with $\kappa, C > 0$.

II) We show with an example that, in general, one cannot reach exactly the constant configurations in finite time. We consider a 2×2 system like the DiPerna one and we prove that if the set of the shocks of a solution at time $t = 0$ is dense then it is dense at each time $t > 0$. In other words if the initial condition has a dense set of shock the reachable set $\mathcal{R}(T)$ does not contain any constant map for each $T > 0$.