On the Boundary Controllability of a Linear System of Hyperbolic Equations

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In this lecture, we want to discuss about the problem of the boundary controllability in finite time for the following linear system of hyperbolic equations

$$\begin{cases} \omega_t + A \cdot \omega_x = 0, & a < x < b,\ t > 0 \\ \omega^+(t,a) = C_a \cdot \omega^-(t,a), & t > 0, \\ \omega^-(t,b) = C_b \cdot \omega^+(t,b) + \Gamma_b \cdot \alpha(t), & t > 0, \\ \omega(0,x) = \omega_0(x), & a < x < b \end{cases} \tag{1}$$

where $\alpha = \alpha(t)$ is the *control*, A is an $n \times n$ diagonal matrix

$$C_a \in \mathbb{R}^{(n-p)\times p},\ C_b \in \mathbb{R}^{p\times(n-p)},\ \Gamma_b \in \mathbb{R}^{p\times\nu},\ \alpha(\cdot) \in \mathbb{R}^{\nu}$$

and we use the following notations

$$\omega^- \doteq \begin{pmatrix} \omega_1 \\ ... \\ \omega_p \end{pmatrix} \in \mathbb{R}^p, \qquad \omega^+ \doteq \begin{pmatrix} \omega_{p+1} \\ ... \\ \omega_n \end{pmatrix} \in \mathbb{R}^{n-p}, \qquad \omega \doteq \begin{pmatrix} \omega_1 \\ ... \\ \omega_n \end{pmatrix} \in \mathbb{R}^n.$$

Where we are assuming that the first p characteristics have negative speeds and the last n-p have positive speeds, namely

$$A \doteq \begin{pmatrix} \lambda_1 & ... & 0 \\ ... & ... & ... \\ 0 & ... & \lambda_n \end{pmatrix}, \quad \lambda_1 < ... < \lambda_p < 0 < \lambda_{p+1} < ... < \lambda_n.$$