Structure analysis of DAEs and general linear methods

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Abstract

Differential algebraic equation (DAEs) are an important tool for modelling real world processes as for instance the simulation of electrical circuits. The modified nodal analysis leads to DAEs of the form

$$A(t)\big(q(x(t),t)\big)' + b\big(x(t),t\big) = 0 \tag{1}$$

where the vector x describes node potentials and the currents of elements like voltage sources and inductors. q contains voltages and fluxes. The successful numerical solution of (1) depends both on a proper formulation of the problem and the choice of appropriate numerical schemes.

The first part of the talk will be devoted to analysing the structure of DAEs (1) with index 2. Using the concept of the tractability index it turns out that the inherent dynamics is determined by a fully implicit index-1 DAE

$$y' = f(y, z'), \qquad z = g(y). \tag{2}$$

A numerically qualified formulation of (1) guarantees that numerical methods, when applied to (1), in fact integrate (2). Methods for DAEs not only have to satisfy the order conditions for ODEs but also a large number of additional conditions as for example order condition for (2), high stage order, stiff accuracy, A- and L-stability, ...

In the second part of the talk we suggest the use of general linear methods (GLMs), a class of integration methods that generalises both linear multistep as well as Runge-Kutta methods. In contrast to Runge-Kutta methods it is possible to have the stage order equal to the order even for diagonally implicit methods.

We will give order condition for GLMs applied to (2) by using DA series and present some first numerical results indicating the good stability properties of these methods.