New Partial Differential Equations Governing the Response-Excitation Joint Probability Distributions of Nonlinear Systems Under General Stochastic Excitation

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Abstract

The problem of determining the probabilistic structure (joint PDFs) of the response of nonlinear dynamical systems under general external stochastic excitation is considered. The method used to address this problem is based on the characteristic functional. As it is known, for any Stochastic Dynamical System, the response-excitation joint Characteristic Functional is governed by an appropriate (always linear!) Functional Differential Equation (FDE), i.e. a differential equation containing both ordinary time derivatives and functional (Volterra) derivatives [Hopf 1952, Beran 1968, Kotulski & Sobczyk 1984, Kotulski 1989]. In this work we first extend some results of Kotulski 1989, showing that apparently all known equations describing the evolution of the probability measure of the dynamical response (i.e. the Fokker-Planck Equation in the case of independent-increments forcing, the Liouville Equation in the case of random initial conditions, and the hierarchy of Moment Equations in the case of any type of stochastic forcing) can be derived directly from the FDE governing the joint Characteristic Functional. Then, we derive new Partial Differential Equations (PDEs) governing the response-excitation (also jointly with any of their derivatives) joint characteristic functions for any set of time values $\left(t_1,t_2,...,t_n\right)$. Further, new PDEs are obtained for the joint probability density functions by applying Fourier transform to the corresponding equations for the characteristic functions. Thus, a new, generic and efficient method to solve the problem of determining the joint response-excitation probabilistic structure of nonlinear dynamical systems, **under any type of stochastic excitation**, is established.

These new PDEs have some peculiarities reflecting the compatibility constraints governing the hierarchy of finite dimensional probability distributions. That is, the supplementary conditions imposed on each one of these PDEs express that the corresponding joint probability density functions (of order $N+1$) should conform with all lower-order response marginals, as well as the excitation marginal. A preliminary method for the numerical solution of these new PDEs is developed and some first numerical results are presented.

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